Simulation of Fine Particulate Matter Dispersion from the Grand Cereals Industry Jos, Nigeria Using a Modified Gaussian Plume Model

Tyoyima John Ayua*, Aondongu Alexander Tyovenda, Emmanuel Vezua Tikyaa, and Shehu Balarabe

***Abstract*—Air pollution in Jos, Nigeria, is concerning due to a variety of anthropogenic sources which expose residents to possible respiratory health risks. Good emission source data and an understanding of all elements connected to air pollution is the foundation for successful pollution abatement. The study established a model for crosswind-integrated concentrations by solving the advection-diffusion equation using the reducible and irreducible techniques and utilizes it to forecast the concentration of PM_X pollutants released from a stack in Jos, Nigeria. Data was collected from the Grand Cereals environment using a Handheld Portable Particle Counter for PM_x with model number CW-HAT 200 for a year. High values of fine particulates recorded in the study are worrisome because they represent a threat to human health. Moreover, the monitored and modeled results evidenced a higher risk for human health in specific points, particularly areas less than 100m away from the stack which is seen as deriving from the stack emissions. Electrostatic precipitators or mist collectors can be installed to reduce the PM_X emissions and its impact on human health. We proposed that the boiler stack be replaced with a taller one and that people living and working near the industry wear a nasal mask to mitigate inhaling dangers. The model performs better in estimating $PM_{2.5}$ concentrations under unstable conditions and is recommended for $PM_{2.5}$ monitoring.**

***Index Terms*—Analytical model, fine particulate pollutant, grand cereals jos, PM_X variation and distribution**

I. INTRODUCTION

The motion of pollutants released from a continuous emission source in the lower atmosphere is a critical issue caused by non-uniform sheared turbulence near rough boundary layers, stability, and complex meteorological conditions such as fog, ambient temperature, prevailing wind speed and direction, and temperature inversion [1, 2]. The shape, evolution, internal structure, and turbulent eddies that characterize plume dispersion all influence plume transport at the lower atmosphere boundary. Despite recent breakthroughs in the theoretical and direct mathematical evaluation of turbulent dispersion in the atmospheric boundary layer (ABL), there remains a need for a simple and rapid technique to forecast the dispersion of turbulent particles in the ABL [3, 4].

The advection-diffusion equation proposed by [5] characterized the dispersion of contaminants in a turbulent environment. The initial and most basic method of modeling air pollution was and continues to be the analytical answer to this equation [6]. In deriving analytical solutions to the advection-Seinfeld diffusion equation, traditional approaches to the air pollution model assumed that eddy diffusivities and wind speed were constant throughout the ABL. The eddies are assumed to be within a set of values parameterized based on available dispersion parameters and distance in the downwind direction in most of the solutions given in the literature, similar to the Gaussian model distribution formula to simulate concentrations close to the source under observation [5, 7]. The observed and predicted findings of several investigations conducted under homogenous stationery and horizontal settings reveal that eddies and wind speed vary with height above the ground level [8].

The assumption of constant wind speed and eddy diffusivity in finding an analytical solution to the advection-diffusion equation was relaxed after statistical analysis based on recent research revealed that downwind distance from the source is responsible for eddy diffusivity [9]. Many eminent researchers have worked hard overtime to solve the advection-diffusion equation in two dimensions under steady-state conditions for a specific type of vertical eddy and wind speed [2, 10, 11].

Crosswind-integrated solutions are valuable in environmental impact and assessment studies because the derived crosswind-integrated concentrations can assist minimize difficulties where the plume spread covers a deep portion of the ABL, altering the height variation with the wind speed [12]. Most analytical solutions in the crosswind direction are found under homogeneous steady-state conditions by using vertical eddies and wind speed as power-law profile functions of vertical height above the ground [3].

The Grand Cereals sector in Jos is critical in terms of creating jobs and meeting human needs to improve the citizens' quality of life. However, the industry's operations emit chemical compounds into the ambient air, which can have a considerable impact on human health depending on the physical and chemical features of these compounds, their concentrations in the air, and the receptor's exposure period. Due to social and economic constraints, improved solutions to control these emissions are not available [9, 13]. Due to the complex nature of inhomogeneous sheared turbulence near rough boundary layers, the stability, and meteorological conditions such as temperature, wind speed and direction, foggy atmosphere, and temperature inversion, the dispersion of air pollutants from a continuous release such as the Grand Cereals industry's stack in Jos in the lower atmosphere is a

Manuscript received May 30, 2022; revised June 30, 2022; accepted July 11, 2022.

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doi: 10.18178/ijesd.2023.14.2.1422

critical problem.

The Grand Cereals plant in Jos primarily processes raw agricultural materials, particularly maize, into semi-finished products like pure grand soya oil, grand flour, grand brabusco, cornflakes, fish and poultry feeds. It is a significant part of Nigeria’s agricultural economy and a well-known source of fine particulate matter emissions, which can have negative effects on human health and the environment. Fine particles, having an aerodynamic equivalent diameter of $2.5 \leq 10$ m, are particularly important. When breathed in the particles can travel deep into the lungs and alveolar tracts, where they are retained [14, 15]. Chronic bronchitis, benign organic dust toxicity syndrome, hyperactive airway disease, chronic asthma, membrane irritation, and even worsening of COVID-19 symptoms have all been linked to fine particulate matter exposure [16–18]. To build successful abatement methods for lowering air pollution, it is critical to have knowledge of emission source data and an understanding of the meteorological conditions that affect air pollution [14, 19].

Because of the economic and temporal constraints involved in in-situ measurements, little or no data on air pollution monitoring and modeling of stack emissions in Jos, Northern Nigeria, and the consequent implications on the environment and its population is unavailable. The goal of this research is to derive an analytical solution by solving the classical Gaussian plume model analytically using reducible and irreducible techniques and applying it to the $PM_{2.5}$ and PM_{10} concentrations data measured from the Grand Cereals industry’s stack in Jos, Nigeria.

To build the model, we made some assumptions like the diffusing pollutants does not have penetrating sources from the downwind distance of the point source, i.e. they are inert, the vertical components of the wind speed and lateral flow of the mean velocity is assumed to be zero, the eddies in the dominant wind direction is negligible and imposed steady state conditions which are eminent conditions from the literature [11, 12, 20].

To overcome the limitations of the existing analytical dispersion models caused by a specific type of vertical eddies and the wind speed associated with turbulent dispersion of ambient air pollutants emitted from a continuous release stack in the ABL, the derived model was presented using the irreducible method which gives a complete solution of the advection-diffusion equation with the wind speed and eddy diffusivities known.

II. MATERIALS AND METHODS

A. *Measurement Site Description*

The Grand Cereals industry is located near the Vom junction on the Bukuru express road in the Bukuru Local Government Area of Plateau State, Jos. Jos is the capital city of Plateau state, and it is located in the northern Guinea Savannah vegetation zone, which is characterized by open forest and tall grasses. Jos is located between 8 °3’ and 10 °28’ north latitude and 08 °20’ and 09 °29’ east longitude. The factory is located at 8 °91’ N and 09 °32’ E, around 1217 meters above sea level (Fig. 1). According to Köppen [21], the height and climate of the area influenced the climate, which is described as wet and dry (Tropical Rainy). The yearly rainfall ranges from 1050 to 1400 mm, and the average annual temperature is around 20 °C. The average wind speed in the research area is between 5.0 and 11.0 meters per second. It has an average temperature of 19.4 °C in the air [22].

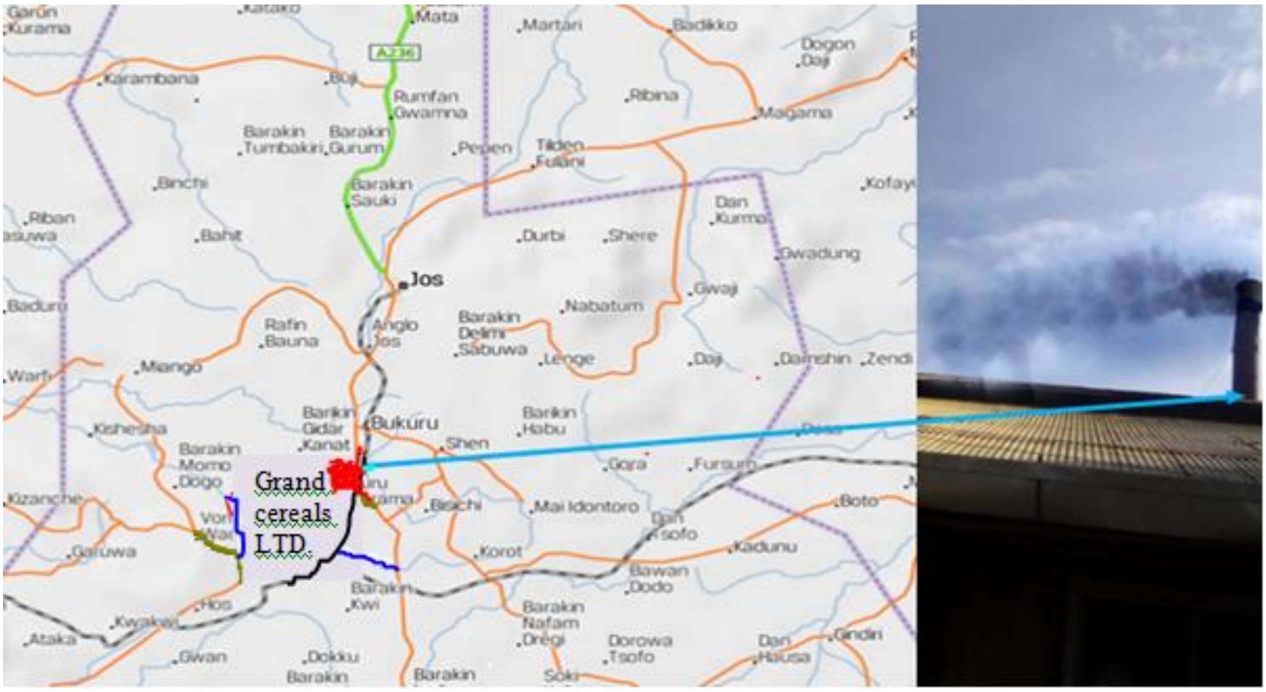

Fig. 1. Map of Jos-Nigeria and its environment showing the study site.

B. *Analytical Solution Procedure of the Advection-Diffusion Equation*

The Fick-theory combined with the continuity equation give rise to the steady-state advection-diffusion equation of the type

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} + V\frac{\partial c}{\partial y} + W\frac{\partial c}{\partial z} = K\nabla^2 C \quad (1)$$

where C= C (x,y,z) means the concentration, K_x, K_y, K_z are the eddy diffusivity components, U, V, W are the wind speed components, x and y are the horizontal distance while z denotes the vertical height above the ground level [11, 23]. Expanding Eq. (1) gives:

$$\frac{\partial c}{\partial t} + \mathrm{U}\frac{\partial c}{\partial x} + \mathrm{V}\frac{\partial c}{\partial y} + \mathrm{W}\frac{\partial c}{\partial Z} = \frac{\partial}{\partial x}\left(K_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial c}{\partial z}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial c}{\partial z}\right) \tag{2}$$

Imposing the assumptions we made from the second to the last paragraph of the introductory section on Eq. (2) we get

$$\mathrm{U}\frac{\partial c}{\partial x} = K_y \frac{\partial^2 c}{\partial y^2} + K_z \frac{\partial^2 c}{\partial z^2} \tag{3}$$

From Eq. (3), if we confined our y and z in the range $0< y < \varepsilon$ and $0 < z < H$, where ε is the distance considered to be far away from the emission source and H is the planetary boundary layer height and x $>$ 0 is the downwind distance.

Following Tyovenda, Ayua, and Sombo [20], expansion and approximation to C(x, y, z) may be more useful as pollutants continue to travel in the air for long distances thus Eq. (3) in an unbounded integral form is:

$$\mathrm{U}\int_{-\infty}^{\infty} \frac{\partial}{\partial x}\mathrm{C}(\mathrm{x, y, z})\, dy = \left[K_y \int_{-\infty}^{\infty} \frac{\partial^2}{\partial y^2}\mathrm{C}(\mathrm{x,y,z}) + K_z \int_{-\infty}^{\infty} \frac{\partial^2}{\partial z^2}\mathrm{C}(\mathrm{x,y,z})\right] dz \tag{4}$$

As y→ ε, $\frac{\partial c}{\partial y} \to 0$ and the first term on RHS vanishes but our steady state conditions may no longer be valid and Eq. (4) can be written as:

$$\frac{\partial c}{\partial z} + \mathrm{U}\int_{-\infty}^{\infty} \frac{\partial}{\partial x}\mathrm{C}(\mathrm{x, z})\, dy = K_z \int_{-\infty}^{\infty} \frac{\partial^2}{\partial z^2}\mathrm{C}(\mathrm{x,z})\, dz \tag{5}$$

Eq. (5) in differential form takes the form of Eq. (6)

$$K \frac{\partial^2 c}{\partial z^2} - \frac{\partial c}{\partial z} - \mathrm{U}\frac{\partial c}{\partial x} = 0 \tag{6}$$

The general Gaussian solution of Eq. (6) as presented by [20, 24] is given in Eq. (7)

$$\mathrm{C(x, y, z)} = \frac{\mathrm{C(X.Z)}}{\sqrt{2\pi}\sigma_\mathrm{y}}\left[\exp\left(-\frac{\mathrm{z}^2}{2\sigma_\mathrm{z}^2}\right)\right]\exp\left(\frac{-\mathrm{y}^2}{2\sigma_\mathrm{y}^2}\right) \tag{7}$$

where C(x, z) is the average pollutant concentration in the crosswind direction and σ_y, σ_z are the dispersion of the pollutant distribution in the x and y directions.

Applying the reducible and irreducible techniques to solve Eq. (6)

We let $\frac{\partial}{\partial x} = \mu,\ and\ \frac{\partial}{\partial z} = \mu'$, and re-write our Eq. (6) as:

$$\left(\mu'^2 - \frac{1}{K}\mu' - \frac{U}{K}\mu\right) C = 0 \tag{8}$$

Observing that Eq. (6) is irreducible; we Let C $= e^{\alpha z+\beta x}$, $\mu C = \beta e^{\alpha z+\beta x}$, $\mu' C = \alpha e^{\alpha z+\beta x}, and$ $\mu'^2 C = \alpha^2 e^{\alpha z+\beta x}$, substituting in our Eq. (8) yields

$$\alpha^2 e^{\alpha z+\beta x} - \frac{1}{K}\alpha e^{\alpha z+\beta x} - \frac{U}{K}\beta e^{\alpha z+\beta x} = 0$$

$$\alpha^2 - \frac{1}{K}\alpha - \frac{U\beta}{K} = 0 \tag{9}$$

Recognizing Eq. (9) as quadratic in α we have:

$$\alpha = \frac{\frac{1}{K} \pm \sqrt{\frac{1}{K^2} + 4\frac{U\beta}{K}}}{2} \tag{10}$$

$$\alpha = \frac{\rho}{2} \pm \frac{\sqrt{\rho^2 + 4\gamma}}{2} \tag{11}$$

With $\rho = \frac{1}{K}$ and $\gamma = \frac{U\beta}{K} = \rho U\beta$
$\Rightarrow\ \alpha = \frac{\rho}{2} \pm \frac{1}{2}\ (\rho^2 + 4\gamma)^{1/2}$

Using Binomial theorem

$$\alpha = \frac{\rho}{2} \pm \left(\frac{\rho^2}{2} + \gamma - \gamma^2 + 2\gamma^3 - 10\gamma^4 + 28\gamma^5 + \ \ldots\right) \tag{12}$$

In general $\alpha_n = \frac{\rho}{2} \pm \sqrt{\frac{\rho^2}{4} + \rho U\beta_n}$, where $\gamma_n = \rho U\beta_n$
Suppose $-\alpha_n = \beta_n$, then we have

$$\alpha_n = -\frac{\rho}{2} \pm \sqrt{\frac{\rho^2}{4} - \rho U\beta_n}$$

$$\alpha_n = -\frac{\rho}{2} \pm i\omega_n$$

$$\omega_n^2 = \rho U\beta_n - \frac{\rho^2}{4}$$

Therefore $C(x,z) = e^{(-\frac{\rho}{2} \pm i\omega_n)x}\ e^{(\frac{\rho}{2} \pm i\omega_n)z} = e^{-\rho U\beta_n x}\ e^{\ i\omega_n z}$

$$C(x,z) = \sum_{i=0}^{n} A_n e^{-\rho U\beta_n x}\{\,\mathrm{Cos}(\omega_n z + B) + \mathrm{Sin}(\omega_n z + \delta_n)\}$$

$$C(x,z) = A_n \sum_{i=0}^{n}[\mathrm{Cos}\,(\omega_n z + B) + \mathrm{Sin}(\omega_n z + \delta_n)\, e^{-\rho U\beta_n x} \tag{13}$$

where A_n, α_n, B, δ_n, and $\omega_n = \rho U\delta_n - \frac{\rho^2}{4}$ are constants to be determine.

Now, following the work of [5, 25] the following boundary conditions are applied to Eq. (3)

1) The diffusing substance is released from a height above the ground level with source strength Q at a point x = (0, h), i.e., UC(x, z) = Q δ (z-h) at x = 0.
2) It is assumed that the ground offered perfect reflections to the diffusing pollutants, i.e., at z $=$ 0, we have $\mathrm{K_z}\frac{\partial \mathrm{C(x,z)}}{\partial \mathrm{z}} = -\mathrm{V_d C} = 0$; where $\mathrm{V_d}$ is the pollutant deposition velocity.
3) The turbulent motion of the pollutants does not go beyond the inversion layer/mixing height

i.e., $K_z \frac{\partial C(x,z)}{\partial z} = 0$ at z = h.

4) Neglecting other penetrating sources, at long distances from the emission point source, i.e., as x, y, z→ ∞, C(x, y, z) = 0.

Applying condition (2) to Eq. (13) gives

$A_n \cos B$= 0

B = 90

Therefore equation (13) becomes

$$C(x,z) = A_n e^{-\rho U \beta_n x} \sum_{i=0}^{n} [(-\mathrm{Sin}\, \omega_n z)(\mathrm{Sin}\, \omega_n z\, Cos\delta_n + Cos\delta_n\, \mathrm{Sin}\, \omega_n z\,)]$$

$$C(x,z) = -A_n e^{-\rho U \beta_n x} \sum_{i=0}^{n} [(\mathrm{Sin}^2\, \omega_n z\, Cos\delta_n) + (\mathrm{Sin}^2\, \omega_n z\, Cos\delta_n)] \quad (14)$$

From the boundary condition (3) at z = h, $C(x, z) = 0$; Eq. (14) becomes

$$0 = 2A_n e^{-\rho U \beta_n x} (\mathrm{Sin}^2\, \omega_n h\, Cos\delta_n)]$$

$(\mathrm{Sin}^2\, \omega_n h\, Cos\delta_n) = 0;$ But $Cos\delta_n \neq 0,$ i.e., $\mathrm{Sin}^2 \omega_n z = 0$, with $\omega_n = \frac{n\pi}{h}$,

We now write our Eq. (14) as:

$$C(x,z) = 2A_n e^{-\rho U \beta_n x} \{\mathrm{Sin}^2 n\pi z\, Cos\delta_n\} \quad (15)$$

Now, applying the first condition Q δ (z-h) = UC(x, z) at $x = 0$ to Eq. (15) yields:

$$C(x,z) = \frac{Q}{hU} 2A_n \int_O^h \mathrm{Sin}^2 n\pi z\, Cos\delta_n dz$$

Where 0 ≤ δ≤ h

$$= 2\frac{Q}{hU} A_n Cos\delta_n \int_O^h \mathrm{Sin}^2 n\pi z\, Cos\delta_n dz$$

$$= 2\frac{Q}{U} A_n Cos\delta_n [\frac{n\pi h}{2} - \frac{\mathrm{Sin}^2\, n\pi h}{4}]$$

$$= \frac{Q}{hU} A_n Cos\delta_n\, (n\pi - \mathrm{Sin}\, n\pi)$$

$$= \frac{Q}{hU} A_n Cos\delta_n n\pi$$

Puting n = 0 gives

$$C(x,z) = \frac{Q}{hU} A_0 = \frac{\lambda}{hU}$$

So We can re-write our Eq. 12 as:

$$C(x,z) = \frac{\lambda}{hU} + \sum_{i=1}^{n} e^{-\rho U \beta_n x} \quad (16)$$

Where λ is the stack emission rate Q multiplied by an arbitrary ith pollutant emission correction factor A_0.

Plugging our original values for $\rho\ and\ \beta_n$ in Eq. (16) gives:

$$C(x,z) = \frac{\lambda}{hU} + \sum_{i=1}^{n} e^{-\left(\frac{U}{2K^2} + \frac{n\pi U}{Kh}\right) x} \quad (17)$$

If we put $\varepsilon_k = \frac{U}{2K^2} + \frac{n\pi U}{Kh}$ and assume that our diffusion depends on the atmospheric stability and height above the ground only then K = $\frac{Z}{U}$ and so

$$\varepsilon_k = \frac{U^3}{2Z^2} + \frac{n\pi U^2}{hZ} \quad (18)$$

Then

$$C(x,z) = \frac{\lambda}{hU} + \sum_{i=1}^{n} e^{-\varepsilon_k x} \quad (19)$$

Substituting Eq. (19) into Eq. (7) gives our modified Gaussian plume model as:

$$\mathrm{C(x, y, z)} = \frac{\lambda + \sum_{i=1}^{n} e^{-\varepsilon_k x}}{\sqrt{2\pi}\sigma_y \mathrm{hu}} \left[\exp\left(-\frac{z^2}{2\sigma_z^2}\right)\right] \exp\left(\frac{-y^2}{2\sigma_y^2}\right)$$

$$\mathrm{C(x, y, z)} = \frac{1}{\sqrt{2\pi}\, \mathrm{hU}\sigma_y} [\lambda + \sum_{i=1}^{n} exp^{-\varepsilon_k x}] \left(exp^{-\frac{z^2}{2\sigma_z^2}} \right)\ exp^{\frac{-y^2}{2\sigma_y^2}} \quad (20)$$

C. Comparison of the Model with Existing Similar Models

The efficacy of the current model Eq. (20) was tested by applying it to the various measurements of the PM_X concentration data that was collected from the Grand cereals industrial stack for a year relative to wind speeds and stability class determination. However, in the offered solution, determining the real form of a numerical equation for U (z) and K_z (x, z) was not attainable. When comparing with some available models already existing, the ground level lower boundary of the cross-wind integrated concentrations were taken at z = 0 similar to [2, 4, 7, 24, 26, 27] who all have different models. Also, different approaches were applied for different wind speed classes and the eddy diffusivity K_z (x, z). In fact, our PM_X dispersion model assumed that K_z(x, z) depends on the atmospheric stability and vertical height above the ground only thereby reducing the complexity faced with other models in solving separately for a definite form of K_z (x, z). Eddy diffusivity was incorporated in our solution of the differential equation which makes our model results novel.

D. Dispersion Parameters Estimation and Stack Characteristics for the Model

The fitted parameters for the running of Eq. (19) are given as follows:

$\lambda = QA_0$, which are defined in Eq. (16);

U is the mean wind speed recorded on data collecting days.

h is the emission stack height in meters, y = 1.5 m = the vertical height above the ground where the measurements were taken.

Z ≈ h_s was estimated following the formulas presented by [14] for open country as:

$$Z = h - \frac{2143}{100U} F_B^{0.75} \tag{21}$$

where F_B is the stack buoyancy flux and is given by

$$F_B = 10\omega_s \left(\frac{\Delta T}{T_s}\right) r_s^2 \tag{22}$$

where ω_s , r_s, T_s are the exit velocity, radius and temperature of the emission stack respectively, and $\Delta T = T_s - T_\theta$; T_θ is the ambient potential temperature [14, 20].

ε_k is computed using Eq. (18).

The sigma power law presented by [3, 25, 28] were used to calculate the dispersion spread parameter σ_y , σ_z as:

$$\sigma_y = ex^{\epsilon} \tag{23}$$

$$\sigma_Z = fx^{\mu} \tag{24}$$

where e, f, ϵ *and* μ are parameters depending on atmospheric stability classes as presented in Table I.

TABLE I: MEAN METEOROLOGICAL DATA SET OF EIGHT CONVECTIVE TEST RUNS

Pasquill Stability Classes	e	f	ϵ	μ
A–B	1.46	0.71	0.01	1.54
C	1.52	0.69	0.04	1.17
D	1.36	0.67	0.09	0.95
E–F	0.79	0.70	0.40	0.67

Source: [28]

where A—Extremely unstable conditions, B- Moderately unstable conditions, C—Slightly unstable conditions, D—Neutral conditions, E—Slightly stable conditions, and F—Moderately stable conditions are the meteorological conditions characterizing the Pasquill turbulence classification in Table I.

The stack characteristics data is also presented in Table II

TABLE II: STACK CHARACTERISTICS AND THE MODELING PARAMETERS

Stack type	Q (m/s)	h (m)	r (m)	T_s°K	Emission factor (A_0)	
					$PM_{2.5}$	PM_{10}
Boiler	4778.9	6.1	0.36	357.2	0.96	1.82

The emission factors (A_0) in Table II were calculated using the formula presented by [29] as in Eq. (25)

$$A_0 = \frac{A_i}{VH} \tag{25}$$

where A_i is the emission of the ith pollutant, V is fuel volume burnt by the boiler, while H is the heating value of the fuel.

Equations 21–24 where used to drive Eq. (20) runs along with the parameters in Tables I and II; near the site data of Annual mean wind speed U and Temperature T were also taken on measurement days which were inputted into the model.

E. PM$_X$ Concentration Measurement

Annual mean PM$_X$ concentrations were measured using a Handheld Portable Particle Counter for PM$_{2.5}$ and PM$_{10}$ from (May 2019–April 2020).

To account for the Grand Cereals stack's contribution to PM$_X$ in the ambient air, the difference between downwind and upwind concentrations was used to reflect the stack emission into the air as shown in Eq. (25)

$$\mathrm{MP_S} = \mathrm{MP_D} - \mathrm{MP_U} \tag{25}$$

where MP_S are the mean concentrations determined from stack contribution only, MP$_D$ is the mean measured concentration at downwind distances and stack contribution and MP$_U$ is the mean concentration measured in the upwind distances only [26]. Repeated readings were taken in every month at specified downwind distances and the mean was taken to compute the required annual mean.

F. Statistical Description of the Model Evaluation

Three statistical tools of Normalized Mean Square Errors N$_{MSE}$, Fractional Bias F$_B$, and Geometric Mean M$_G$ were applied to the measured and predicted values as pairs to evaluate the performance of the model. A good model should have values of N$_{MSE}$ and F$_B$ close to zero while that of M$_G$ close to unity. These statistics are defined as follows:

$$\mathrm{N_{MSE}} = \frac{\overline{(C_m - C_p)^2}}{(\overline{C_m})(\overline{C_p})} \tag{26}$$

$$F_B = \frac{2(\overline{C_m} - \overline{C_p})}{(\overline{C_m}) + \overline{C_p}} \tag{27}$$

$$M_G = e^{-(\overline{\ln C_p} - \overline{\ln C_m})} \tag{28}$$

where C_m and C_p refers to the measured and observed concentrations of the PM$_X$ respectively [1, 25, 30].

III. RESULTS AND DISCUSSION

Annual mean concentrations of the PM$_X$ pollutants measured/modeled in µg m^{-3} and how the concentrations of these pollutants vary with downwind distance and months of the year are presented in Table III and Figs. 2-4. Also presented in Fig. 4 is the comparison of the monthly means measured/modeled with the World Health Organization/National Ambient Air Quality Standards (WHO/NAAQS) to gauge air quality within the industry and its environs. Table IV presented an analysis of the statistical tools for model evaluation.

TABLE III: ANNUAL MEAN CONCENTRATIONS OF PMX (µG M^{-3}) AS COMPUTED FROM THE MEASURED/MODELED MONTHLY MEANS IN THE GRAND CEREALS INDUSTRY, JOS

	Site			
	Grand Cereals Nig. Ltd.			
	Measured	Modeled		
X (m)	PM$_{2.5}$	PM$_{10}$	PM$_{2.5}$	PM$_{10}$
10.0	48.33± 2.71	75.03 ± 2.20	40.92	77.42

20.0	33.39 ± 4.10	55.83 ± 3.20	41.35	78.38
30.0	29.33 ± 3.60	52.52 ± 2.90	31.44	59.61
40.0	25.69 ± 2.60	50.52 ± 2.10	24.73	46.88
50.0	23.32 ± 1.90	44.05 ± 1.90	20.25	38.38
60.0	20.02 ± 1.20	30.05 ± 0.90	17.10	32.42
70.0	18.90 ± 3.30	29.92 ± 1.70	14.79	28.04
80.0	16.60 ± 3.20	28.32 ± 1.20	13.02	24.69
90.0	14.72 ± 2.20	29.81 ± 3.60	11.63	22.06
100.0	15.12 ± 4.20	25.81 ± 3.00	10.51	19.93
110.0	14.04 ± 1.40	25.92 ± 1.85	9.59	18.19
120.0	13.04 ± 1.90	24.82 ± 1.90	8.82	16.72
130.0	11.65 ± 1.20	28.80 ± 2.96	8.17	15.48
140.0	9.35 ± 3.20	16.80 ± 2.26	7.60	14.41
150.0	9.98 ± 2.80	16.49 ± 3.09	7.11	13.49
160.0	6.11 ± 3.60	22.29 ± 4.29	6.69	12.68
170.0	4.92 ± 3.20	16.25 ± 2.62	6.31	11.96
180.0	4.72 ± 3.20	13.25 ± 2.00	5.97	11.32
190.0	4.98 ± 2.90	19.99± 2.70	5.67	10.75
200.0	4.58 ± 4.20	14.86 ± 3.60	5.40	10.23

Table III shows the yearly mean PM_X concentrations measured and modeled. All measured PM_X concentrations exceeded the modeled values at all downwind distances except at 20.0 m, where the opposite was true. This could be because fine particulates in the research area's ambient air come from a variety of sources other than the stack. In the downwind direction, higher PM_X values were collected within 20.0–40.0 m from the stack. This means that receptors in this area are more vulnerable to the risks of respiratory disorders caused by inhaling these specific materials. As the distance from the emission stack rises, measurement and projected values (Table III) converge as distance from the emission stack rises, indicating that the model is a convergent series that is perfect for estimating air pollution emission and dispersion from a single point source. The standard errors were added to the mean of the measured values to demonstrate their level of variance over time and space.

Fig. 2–4 show remarkable variations which may be linked to differences in climatic elements such as wind speed and direction, temperature, relative humidity, and the month's atmospheric stability class, which are all key determinants of air pollution dispersion in the atmosphere. Jos has more unstable cases, according to the findings. With two key elements, wind strength and air stability, atmospheric stability impacts vertical air motions and pollution dispersion [9, 31]. Pollutant concentrations are higher when winds are light or quiet than when winds are strong [24]. These could explain why certain months have higher pollution concentrations than others. In Jos, the stability conditions were largely neutral and near-stable. Conclusively, the weather was chilly. Because cold air is thicker than it corresponding warm counterpart, chilled air at the surface descends off slopes into nearby lowlands and valleys. These thicker surface inversions will not diffuse out as rapidly after daybreak, as one might assume.

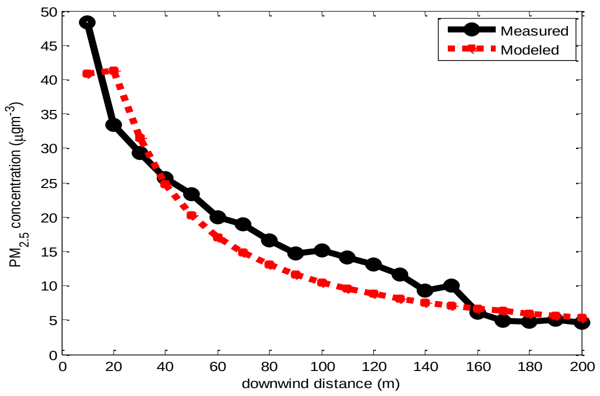

Fig. 2. Comparison of annual mean concentrations of $PM_{2.5}$ measured/modeled with downwind distance in Grand Cereals Nig. Ltd., Jos in May 2019–April 2020.

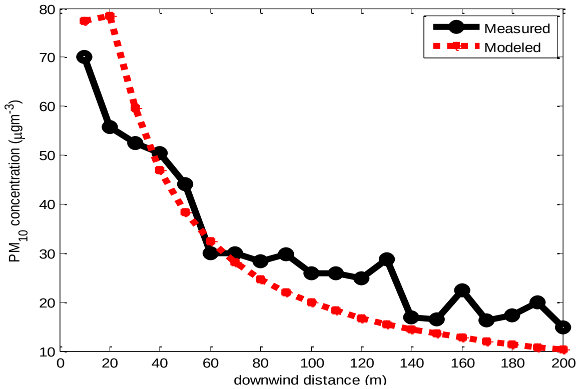

Fig. 3. Comparison of annual mean concentrations of PM_{10} measured/modeled with downwind distance in Grand Cereals Nig. Ltd., Jos in May 2019–April 2020.

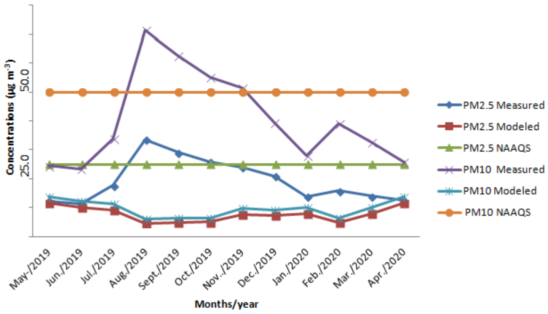

Fig. 4. Variations of monthly mean concentrations of $PM_{2.5}$, and PM_{10}, measured/modeled with months/comparison with NAAQS in grand cereals jos.

Monthly mean $PM_{2.5}$ and PM_{10} concentrations measured/modeled in the Grand Cereals industrial Jos fluctuate greatly over months/seasons of the year, as shown in Fig. 4. Rainfall, wind speed, and direction are all elements that influence these variances. From August 2019 to January 2020, PM_X concentrations were higher (onset of the dry season). The reason may be because, throughout the study period, rainfall and washout were reduced, whereas dust re-suspension was increased.

When comparing the mean measured/modeled concentrations to the NAAQS/WHO standard limit values of 25.0 g m^{-3} for $PM_{2.5}$ and 50.0 g m^{-3} for PM_{10}, the NAAQS/WHO standard limit values of 25.0 g m^{-3} for $PM_{2.5}$ and 50.0 g m^{-3} for PM_{10} were used. The concentrations of PM_X were lower, as shown by the mean standard in Fig. 4. However, measured PM_X concentrations in August, September, October, and November surpassed these acceptable levels, posing a health concern to those living and working near the industry's location.

TABLE IV: STATISTICAL EVALUATION OF THE MODEL PERFORMANCE

Statistical Tool		N_{MSE}	M_G	F_B
PM_X	$PM_{2.5}$	0.88	0.75	0.85
	PM_{10}	2.14	0.66	1.18

Table IV shows that the presented model performs better in predicting the concentrations of $PM_{2.5}$ in μg m^{-3} than PM_{10}. The reason could not be far fetch from the fact that the PM_{10} originate from other variety of sources like household cooking, motor vehicles, street dust, and storms just to mention a few that were inevitably captured from the measurement other than the stack characteristics upon which the model was built. The model can, therefore, be better applied for $PM_{2.5}$ prediction where in situ measurement is impossible. Further research may re-work on the model emission factors to improve its performance for the case of PM_{10}.

IV. CONCLUSION

A derived analytical model for forecasting crosswind-integrated concentrations is presented to examine the turbulent dispersion of PM_X produced from an elevated stack. The model assumed that the diffusion depends on the atmospheric stability and height above the ground only. The analytical solution of the advection-diffusion equation obtained using our methods is found to be close to some of the solutions reported in the literature using traditional techniques.

The model was applied to a one-year data acquired using a Handheld Portable Particle Counter for $PM_{2.5}$ and PM_{10} fine particulate matter PM_X measurement. The results were analyzed using statistical tools and also compared to WHO/NAAQS limit values to determine the possible degree of health consequences of these pollutants on receptors within the Grand Cereals facility and its adjacent areas. The analytical model performs better in estimating $PM_{2.5}$ concentrations in μg m^{-3} under unstable conditions when tested with relevant statistical tools. We proposed that the boiler stack be replaced with a taller one, and that people living and working near the industry wear a nasal mask to protect themselves from the harmful effects of inhaling these particulates. Particularly, increasing the stack height to about 70.0 m will reduce the ground level concentrations below limit levels as experimented with the model.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

AUTHOR CONTRIBUTIONS

T. J. Ayua conducted the research, developed the model used and wrote the first draft of the manuscript; A. A. Tyovenda supervised the experimental procedures; E. V. Tikyaa analyzed the statistical tools; S. Balarebe wrote the code for the model; all the authors had approved the final version of the manuscript.

ACKNOWLEDGMENTS

We sincerely appreciated the kindness of the management of the Grand Cereals industry, Jos especially Mr. Aeron and Eng. Dafeng, D. for giving us the stack characteristics data and also granting us access to carry out this assessment.

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