Discontinuous Galerkin Method for the Air Pollution Model

Xijian Wang and Lite Zhao

Abstract—In this paper we present the discontinuous Galerkin method to solve the problem of the two-dimensional air pollution model. The resulting system of ordinary differential equations is called the semidiscrete formulation. We show the existence and uniqueness of the ODE system and provide the error estimates for the numerical error.

Index Terms—Air pollution model, discontinuous galerkin method, error estimate

I. INTRODUCTION

Air pollution is the introduction of chemicals, particulate matter, or biological materials that cause harm or discomfort to humans or other living organisms, or cause damage to the natural environment or built environment, into the atmosphere. The basic technology for analyzing air pollution is through the mathematical models and numerical methods for predicting the transport of air pollutants in the lower atmosphere[1]-[5]. Different air pollution models have been developed in the last decades by the National Environmental Research Institute (<http://www.dmu.dk/en/air/models/>). In the present paper we consider the following Danish Eulerian model[2], [4], [5].

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(cu) + \frac{\partial}{\partial y}(eu) - \frac{\partial}{\partial x}(k_x \frac{\partial u}{\partial x}) - \frac{\partial}{\partial y}(k_y \frac{\partial u}{\partial y}) &= f(u), \\ f(u) &= -(k_1 + k_2)u + E + Q(u), \\ u(x, y, 0) &= u_0(x, y), \quad (x, y) \in \Omega, \\ u(x, y, t)|_{\partial\Omega} &= 0, \quad t \in [0, T]. \end{aligned} \quad (1)$$

The different quantities involved in the mathematical model have the following meaning:

- 1) the concentration is denoted by u ;
- 2) c and e are wind velocities;
- 3) k_x and k_y are diffusion coefficients;
- 4) the emission source is described by E ;
- 5) k_1 and k_2 are constant deposition coefficients;
- 6) the chemical reaction is denoted by Q .

Meanwhile, we give the following assumptions:

- 1) $u \in H_0^1(\Omega) \cap H^3(\Omega)$, $u_t, u_{tt} \in L^2(\Omega)$;
- 2) $Q(u)$ satisfy the Lipschitz condition;
- 3) $0 < k_* \leq \min\{|k_x|, |k_y|\} \leq \max\{|k_x|, |k_y|\} \leq k^*$,
 $0 < c_* \leq \min\{|c|, |e|\} \leq \max\{|c|, |e|\} \leq c^*$,
 k_*, k^*, c_*, c^* are constants.

A general description of the Danish Eulerian Model and its numerical treatment is given in [5]-[7]. Research on the finite difference method and finite volume element method for this air pollution model already has good results [8]-[11]. In this article, we use the discontinuous Galerkin method (DG method) to analyse and solve the air pollution model.

DG methods in mathematics form a class of numerical methods for solving partial differential equations. They have recently gained popularity due to many of their attractive properties, refer to [12]-[20]. First of all, the flexibility of the methods allows for general non-conforming meshes with variable degree of approximation. This makes the implementation of h-p adaptivity for DG easier than that for conventional approaches. Moreover, the DG methods are locally mass conservative at the element level. In addition, they have less numerical diffusion than most conventional algorithms, thus are likely to offer more accurate solution for at least advection-dominated transport problems. They handle rough coefficient problems and capture the discontinuity in the solution very well by the nature of discontinuous function space. Furthermore, the DG methods are easier to implement than most traditional finite element methods. The trial and test spaces are easier to construct than conforming methods because they are local.

The paper is organized as follows: In Section 2, the variational formulation of the DG method is stated. And we show the existence and uniqueness of the resulting ordinary differential equations system. Finally we provide the error estimates for the numerical error in Section 3.

II. SEMIDISCRETE FORMULATION

In this section, we approximate the solution $u(t)$ by a function $U_h(t)$ that belongs to the finite-dimensional space $\mathcal{D}_k(\mathcal{E}_h)$ for all $t \geq 0$. The solution U_h is referred to as the semidiscrete solution. In what follows, we assume that $s > \frac{3}{2}$. We introduce a bilinear form $J_0^{\sigma_0, \beta_0} : H^s(\mathcal{E}_h) \times H^s(\mathcal{E}_h) \rightarrow \mathbb{R}$ that penalize the jump of the function values:

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$$J_0^{\sigma_0, \beta_0}(w, v) = \sum_{e \in \Gamma_h \cup \partial\Omega} \frac{\sigma_e^0}{|e|^{\beta_0}} \int_e [w][v].$$

The parameter σ_e^0 is called penalty parameter. It is nonnegative real number. The power β_0 is positive number. $|e|$ simply means the length of e . We now define the DG bilinear form $a_\epsilon : H^s(\mathcal{E}_h) \times H^s(\mathcal{E}_h) \rightarrow \mathbb{R}$

$$\begin{aligned} a_\epsilon(w, v) = & \sum_{E \in \mathcal{E}_h} \int_E (k_x \frac{\partial w}{\partial x} \frac{\partial v}{\partial x} + k_y \frac{\partial w}{\partial y} \frac{\partial v}{\partial y}) \\ & - \sum_{e \in \Gamma_h} \int_e (\{k_x \frac{\partial w}{\partial x} \vec{n}_1\} + \{k_y \frac{\partial w}{\partial y} \vec{n}_2\})[v] \\ & - \epsilon \sum_{e \in \Gamma_h} \int_e (\{k_x \frac{\partial v}{\partial x} \vec{n}_1\} + \{k_y \frac{\partial v}{\partial y} \vec{n}_2\})[w] \\ & + J_0^{\sigma_0, \beta_0}(w, v). \end{aligned}$$

The bilinear form a_ϵ contains another parameter ϵ that may take the value -1, 0 or 1. a_ϵ is symmetric if $\epsilon = -1$ and it is nonsymmetric otherwise.

This bilinear form yields the following energy seminorm:

$$\|v\|_\epsilon = (\sum_{E \in \mathcal{E}_h} \|D^{1/2} \nabla v\|_{L^2(E)}^2 + \sum_{e \in \Gamma_h} \frac{\sigma_e^0}{|e|^{\beta_0}} \|[v]\|_{L^2(e)}^2)^{1/2}.$$

Second, the convection term $\frac{\partial}{\partial x}(cu) + \frac{\partial}{\partial y}(eu)$ is approximated by an upwind discretization. Let us denote the upwind value of a function w by w^{up} . We recall that $\begin{pmatrix} \vec{n}_1 \\ \vec{n}_2 \end{pmatrix}$ is a unit normal vector pointing from E_e^1 to E_e^2 :

$$w^{up} = \begin{cases} w|_{E_e^1}, & \text{if } c\vec{n}_1 + e\vec{n}_2 \geq 0 \\ w|_{E_e^2}, & \text{if } c\vec{n}_1 + e\vec{n}_2 \leq 0 \end{cases} \quad \forall e = \partial E_e^1 \cap \partial E_e^2.$$

Let

$$\begin{aligned} b(c, e; w, v) = & - \sum_{E \in \mathcal{E}_h} \int_E (cw \frac{\partial v}{\partial x} + ew \frac{\partial v}{\partial y}) \\ & + \sum_{e \in \Gamma_h} \int_e (c\vec{n}_1 w^{up}[v] + e\vec{n}_2 w^{up}[v]). \end{aligned}$$

The general semidiscrete DG variational formulation of our problem (1) is as follows: Find $U_h \in L^2(0, T; \mathcal{D}_k(\mathcal{E}_h))$, such that $\forall t > 0, \forall v \in \mathcal{D}_k(\mathcal{E}_h)$,

$$\begin{aligned} (\frac{\partial U_h}{\partial t}, v)_\Omega + a_\epsilon(U_h(t), v) + b(c, e; U_h(t), v) &= L(U_h(t), v), \quad (2) \\ (U_h(0), v)_\Omega &= (u_0, v)_\Omega, \end{aligned}$$

where the form L is

$$L(w, v) = \int_\Omega f(w)v.$$

The next lemma establishes the consistency between the model problem and the variational formulation.

Lemma1: Assume that the weak solution u of problem (1) belongs to $H^1(0, T; H^2(\mathcal{E}_h))$, then u satisfies the variational problem (2).

Proof: Let v be a test function in $\mathcal{D}_k(\mathcal{E}_h)$. We multiply by $v|_E$ and integrate by parts on one element $E \in \mathcal{E}_h$, and use Green's theorem:

$$\begin{aligned} (\frac{\partial u}{\partial t}, v)_E - \int_E (-k_x \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} - k_y \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} + cu \frac{\partial v}{\partial x} + eu \frac{\partial v}{\partial y}) + \\ \int_{\partial E} (-k_x \frac{\partial u}{\partial x} \vec{n}_1 v - k_y \frac{\partial u}{\partial y} \vec{n}_2 v + c u \vec{n}_1 v + e u \vec{n}_2 v) = \int_E f(u)v \end{aligned}$$

Summing over all elements and using the regularity of the exact solution, we obtain

$$\begin{aligned} (\frac{\partial u}{\partial t}, v)_\Omega - \sum_{E \in \mathcal{E}_h} \int_E (-k_x \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} - k_y \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} + cu \frac{\partial v}{\partial x} + eu \frac{\partial v}{\partial y}) + \\ \sum_{e \in \Gamma_h} \int_e (-\{k_x \frac{\partial u}{\partial x} \vec{n}_1\}[v] - \{k_y \frac{\partial u}{\partial y} \vec{n}_2\}[v] + c u \vec{n}_1[v] + e u \vec{n}_2[v]) + \\ \epsilon \sum_{e \in \Gamma_h} \int_e (-\{k_x \frac{\partial v}{\partial x} \vec{n}_1\}[u] - \{k_y \frac{\partial v}{\partial y} \vec{n}_2\}[u]) + \sum_{e \in \Gamma_h} \frac{\sigma_e^0}{|e|^{\beta_0}} \int_e ([u][v]) \\ = \int_\Omega f(u)v. \end{aligned}$$

Since $u_{up} = u$, we clearly have our result.

A. Existence and Uniqueness of the Solution

Because of the lack of continuity constraints between mesh elements for the test functions, the basic functions of $\mathcal{D}_k(\mathcal{E}_h)$ have a support contained in one element. We write

$$\mathcal{D}_k(\mathcal{E}_h) = \text{span}\{\phi_i^E : 1 \leq i \leq N_{loc}, E \in \mathcal{E}_h\}$$

With

$$\phi_i^E(x) = \begin{cases} \tilde{\phi}_i \circ F_E(x), & x \in E, \\ 0, & x \notin E. \end{cases}$$

In 2D, we have $\hat{\phi}(\hat{x}, \hat{y}) = \hat{x}^I \hat{y}^J, I + J = i, 0 \leq i \leq k$.

$$N_{loc} = \frac{(k+1)(k+2)}{2}.$$

using the global basis functions, we can expand the semidiscrete solution $\forall t \in (0, T), \forall (x, y) \in \Omega$,

$$U_h(t, x, y) = \sum_{E \in \mathcal{E}_h} \sum_{i=1}^{N_{loc}} \xi_i^E(t) \phi_i^E(x, y). \quad (3)$$

The degrees of freedom ξ^E 's are functions of time. Let N_{el} denote the number of elements in the mesh. We can rename the basis functions and the degrees of freedom such that

$$\{\phi_i^E : 1 \leq i \leq N_{loc}, E \in \mathcal{E}_h\} = \{\tilde{\phi}_j : 1 \leq j \leq N_{loc} N_{el}\},$$

$$\{\xi_i^E : 1 \leq i \leq N_{loc}, E \in \mathcal{E}_h\} = \{\tilde{\xi}_j : 1 \leq j \leq N_{loc} N_{el}\}.$$

Plugging (3) into the variational problem (2) yields a linear system of ordinary differential equations as follows:

$$M \frac{d\tilde{\xi}}{dt}(t) + (A + B)\tilde{\xi} = G(\tilde{\xi}),$$

$$M \tilde{\xi}(0) = \tilde{U}_0.$$

The matrices M, A are called the mass and stiffness matrices, and they are defined by

$$\forall 1 \leq i, j \leq N_{loc} N_{el}, \quad M_{ij} = (\tilde{\phi}_j, \tilde{\phi}_i)_\Omega, \quad A_{ij} = a_\epsilon(\tilde{\phi}_j, \tilde{\phi}_i).$$

The matrix B results from the convective term, and the vector $G(\tilde{\xi})$ depends on the vector solution

$$\forall 1 \leq i, j \leq N_{loc} N_{el}, \quad (B)_{ij} = b(c; e; \tilde{\phi}_j, \tilde{\phi}_i),$$

$$\forall 1 \leq i \leq N_{loc} N_{el}, \quad (G)_i = L(\tilde{\xi}; \tilde{\phi}_i).$$

Since the matrix M is invertible and the vector function $G(\tilde{\xi})$ is Lipschitz with respect to $\tilde{\xi}$, there exists unique solution to the variational problem (2).

B. Error Estimates

In this section, we first present the Gronwall's inequalities [21], which are important tools for analyzing time-dependent problems.

Lemma2 (Continuous Gronwall inequality): Let f, g, h be piecewise continuous nonnegative functions defined on (a, b) . Assume that g is nondecreasing. Assume that there is a positive constant C independent of t such that

$$\forall t \in (a, b), \quad f(t) + h(t) \leq g(t) + C \int_a^t f(s) ds.$$

Then

$$\forall t \in (a, b), \quad f(t) + h(t) \leq e^{C(t-a)} g(t).$$

Now we state a priori error estimates for the semidiscrete scheme [22].

Theorem1: Assume that the solution u to problem [1] belongs to $H^1(0, T; H^2(\mathcal{E}_h))$ and that u_0 belongs to $H^s(\mathcal{E}_h)$ for $s > 3/2$. Assume that $\beta_0 \geq 1$. In the case of SIPG and IIPG, assume that σ_e^0 is sufficiently large for all e . Then, there is a constant C independent of h such that

$$\|u - U_h\|_{L^\infty(L^2(\Omega))} + \left(\int_0^T \|u(t) - U_h(t)\|_\epsilon^2 dt \right)^{1/2}$$

$$\leq Ch^{\min(k+1, s)-1} (\|u\|_{H^1(0, T; H^s(\mathcal{E}_h))} + \|u_0\|_{H^s(\mathcal{E}_h)}).$$

Proof: We omit some details which is similar to the proof of Theorem 3.4. ([13]). We write $u - U_h = \rho - \chi$ with

$\rho = u - \tilde{u}$ and $\chi = U_h - \tilde{u}$. The function $\tilde{u} \in \mathcal{D}_k(\mathcal{E}_h)$ is an approximation of u that satisfies good error bounds. The error equation is satisfied for all v in $\mathcal{D}_k(\mathcal{E}_h)$:

$$\left(\frac{\partial \chi}{\partial t}, v \right)_\Omega + a_\epsilon(\chi, v) + b(c, e; \chi, v) = \left(\frac{\partial \rho}{\partial t}, v \right)_\Omega$$

$$+ a_\epsilon(\rho, v) + b(c, e; \rho, v) + (f(U_h) - f(u), v)_\Omega.$$

Now, by choosing $v = \chi$ and using the coercivity of a_ϵ , we obtain

$$\frac{1}{2} \frac{d}{dt} \|\chi\|_{L^2(\Omega)}^2 + \kappa \|\chi\|_\epsilon^2 + b(c; e; \chi, \chi) \leq \left(\frac{\partial \rho}{\partial t}, \chi \right)_\Omega$$

$$+ a_\epsilon(\rho, \chi) + b(c, e; \rho, \chi) + (f(U_h) - f(u), \chi)_\Omega.$$

We use Green's formula and the fact that $\nabla \cdot \begin{pmatrix} c \\ e \end{pmatrix} = 0$, then

we have

$$\sum_{E \in \mathcal{E}_h} \int_E \begin{pmatrix} c \\ e \end{pmatrix} \chi \cdot \nabla \chi = \frac{1}{2} \sum_{E \in \mathcal{E}_h} \int_E \begin{pmatrix} c \\ e \end{pmatrix} \cdot \nabla \chi^2$$

$$= \frac{1}{2} \sum_{E \in \mathcal{E}_h} \int_{\partial E} \begin{pmatrix} c \\ e \end{pmatrix} \cdot \begin{pmatrix} \vec{n}_1 \\ \vec{n}_2 \end{pmatrix} \chi^2$$

$$= \frac{1}{2} \sum_{e \in \Gamma_h} \int_e (c \vec{n}_1 + e \vec{n}_2) [\chi^2].$$

Thus we obtain

$$b(c, e; \chi, \chi)$$

$$= - \sum_{E \in \mathcal{E}_h} \int_E \begin{pmatrix} c \\ e \end{pmatrix} \chi \cdot \nabla \chi + \sum_{e \in \Gamma_h} \int_e (c \vec{n}_1 + e \vec{n}_2) \chi^{up} [\chi]$$

$$= \sum_{e \in \Gamma_h} \int_e (c \vec{n}_1 + e \vec{n}_2) (\chi^{up} [\chi] - \frac{1}{2} [\chi^2])$$

$$= \sum_{e \in \Gamma_h} \int_e (c \vec{n}_1 + e \vec{n}_2) (\chi^{up} [\chi] - \{\chi\} [\chi])$$

$$= \frac{1}{2} \sum_{e \in \Gamma_h} \int_e |c \vec{n}_1 + e \vec{n}_2| [\chi]^2 \geq 0.$$

We now bound each term in $b(c, e; \rho, \chi)$. Using Cauchy-Schwarz's and Young's inequalities, we have

$$\sum_{E \in \mathcal{E}_h} \int_E \begin{pmatrix} c \\ e \end{pmatrix} \rho \cdot \nabla \chi \leq C \sum_{E \in \mathcal{E}_h} \|\rho\|_{L^2(E)} \|\nabla \chi\|_{L^2(E)}$$

$$\leq \frac{\kappa}{8} \|\chi\|_\epsilon^2 + C \|\rho\|_{L^2(\Omega)}^2$$

and

$$\begin{aligned} & \sum_{e \in \Gamma_h} \int_e (\bar{c}n_1 + \bar{c}n_2) \chi^{up} [\chi] \\ & \leq \sum_{e \in \Gamma_h} \left\| \bar{c}n_1 + \bar{c}n_2 \right\|_{0,e}^{\frac{1}{2}} \left\| \chi \right\|_{0,e} \left\| \bar{c}n_1 + \bar{c}n_2 \right\|_{0,e}^{\frac{1}{2}} \rho_* \\ & \leq \frac{1}{4} \sum_{e \in \Gamma_h} \left\| \bar{c}n_1 + \bar{c}n_2 \right\|_{0,e}^2 \left\| \chi \right\|_{0,e}^2 + C \sum_{e \in \Gamma_h} \left\| \rho^{up} \right\|_{L^2(e)}^2. \end{aligned}$$

Finally, we bound the nonlinear source term, using the Lipschitz property:

$$\begin{aligned} \int_{\Omega} (f(U_h) - f(u)) \chi & \leq C \|U_h - u\|_{L^2(\Omega)} \|\chi\|_{L^2(\Omega)} \\ & \leq C \|\chi\|_{L^2(\Omega)}^2 + C \|\rho\|_{L^2(\Omega)}^2. \end{aligned}$$

The other terms are identical to the ones in the proof of Theorem 2.13 and 3.4 ([13]). Then the main result is obtained by combining all bounds and using Gronwall's inequality of **Lemma2**.

We can choose any of the time discretizations such as backward Euler and forward Euler and some that are of high order such as Crank-NicThe author wishes to thank the financial support from the Erasmus Mundus Scholarship of the European Union during his visiting study in Europe and thus has an opportunity to join in the discussion of Discontinuous Galerkin method in Johannes Kepler University, Linz, Austria in 2009.

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