

Experimental Analytical Simulation Method in Landfill Geomembrane Liner Design

Hessamallah Shakeri, Mohssen Shoeybi, and Jonathan L. Salvacion

Abstract—The Environmental concerns lead to consider geomembrane liner as a major parameter in landfill design for inhibition of their leachate to the soil and groundwater migration. This study is conducted to determine the effect of physico-mechanical properties of geomembrane material on appropriate thickness in landfill liner with consider to landfill geometry. Experimental data (i.e. Tensile strength and friction angle) by using analytical simulation method with association statistical analysis based on conventional theory was investigated and validated with finite element modeling using ANSYS Software. In addition, effect of safety factor contribution in landfill site condition and friction angle of geomembrane was studied. Moreover, regression and correlation analysis to describe the relationship between the site condition variables and corresponding thickness were performed and equations of fitted models were determined.

Index Terms—Geomembrane, thickness, modeling, landfill.

I. INTRODUCTION

The growth of solid waste generation has caused a range of adverse socio-economic, environmental and health impacts. Due to economic advantages, the sanitary landfilling method is widely accepted as a solid waste treatment. Besides its economic advantages, landfilling minimizes environmental insults and other inconveniences, and allows waste to decompose under controlled conditions until its eventual transformation into relatively inert, stabilized material [1]. The utilization of synthetic liners geomembrane for the recent years has seen increasing. This is mainly due to their easy availability and low volume consumption.

Geomembranes are kind of geosynthetic impermeable material used extensively as liner system in sanitary landfill sites. The landfill liners are constructed from various plastic materials, including polyvinyl chloride (PVC) and high-density polyethylene (HDPE) and the preferred material used in MSW and secured landfills is HDPE [2]. Flexible plastic aluminum laminates (FPAL) is a multi-component plastic food packaging material that was recycled by simple physico-mechanical recycling process methodology which produced Recycled Plastic Aluminates [3]. Based on the study conducted by [3], the characterization of properties of recycled plastic aluminum laminate have great potential as liner system for sanitary landfill and due to its properties are comparable with HDPE liners. Geomembrane properties must be known in order to estimate the maximum available tensile strength of the liner [4]. One of the parameters of

geomembrane materials is durability which is a function of its thickness. The required minimum geomembranes thickness as specified in regulations varies from country to country but is usually related to an implied longer service life [5] and should be a function of design which implies specific site information, and consideration [6]. Thickness is a basic property of geomembranes that is used for general identification and classification of these materials [7] and it is used in all phases of production and lifetime of geomembranes including manufacturing and design [8]. Also mechanical properties and resistance to transmission of fluids for geomembrane material are affected by thickness [9]-[10]. For determination of thickness; is required to calculate the numerical values of properties, such as tensile strength and interface friction angle between liner and soil.

Significant of the tensile strength capacity in geomembrane materials is to determining the stability of a landfill liner system because the liner could rupture due to the existence of tensile stresses resulted by the weight of waste on the materials. In addition, the long term performance of geomembrane materials in terms of leachate resistance and creep resistance is also a function of tensile stress [11]. Therefore, it is important to evaluate the tension and tensile strength in geomembrane induced by weight of the overburden material. Significant number of investigations had been conducted in liner characterization, the vast majority of them focused on evaluation of friction angles. For instance, several interfaces of landfill liner components with conducting direct shear test is investigated in [12] and results showed that interface shear capacity is enhanced by modifying the geomembrane surface texture. Therefore, it is important to evaluate the tension in geomembrane induced by downward dragging of cover soils due to friction forces on geomembrane materials.

A new analytical method that considers displacement compatibility in estimating the tension in geosynthetic materials in a landfill liner system is introduced in [13]. Geomembranes are manufactured with thicknesses ranging from 1 mm to 3 mm [14]. Thickness optimization of a structure means determination of thickness field in order to get some optimum result which can be minimum weight, stress and strain related quantities [15]. Landfill geometry site dimensions are involved volume of solid waste, available land area; maximum depth and slope angle are considered as important parameters in landfill liner design. Hence, effect of these parameters in geomembrane thickness became the inspiration of this study. For a modern municipal solid waste landfill a liner system is a mandated component. In general, a liner system is composed of barrier, drainage, and cover layers which fulfill the purpose of separating buried waste or leachate from the underlying soil and groundwater [13]. This

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Authors are with Graduate Studies School, Mapua Institute of Technology, Intramuros, Manila, Philippine (e-mail: hshakeri@mymail.mapua.edu.ph, mshoeybi@mymail.mapua.edu.ph, jlsalvacion@mapua.edu.ph).

study considered single geomembrane-clay liner with sand layer as leachate collection system in landfill. The rapid growth of urbanization and economic development increases the municipal solid waste generation. Hence, the disposal of municipal solid waste and availability of land fill site area is the common problem in all developing countries in Asia [16]. Thus, the need of high capacity landfill with appropriate life time is sensible.

The main objective of this study is to determine the effect of landfill site condition in appropriate thickness of geomembrane with the experimental analytical and statistic method in terms of capacity of landfill and liner material reduction in order to achieve the best performance and stability.

II. MATERIAL AND METHODOLOGY

A. Material

Two kinds of liner material were investigated in this study. HDPE is a common geomembrane liner that is used widely as landfill liner and recycled plastic aluminum with different percentage of organo clay (i.e. 100 percent of FPAL for Type 1, 99% of FPAL and 1% organoclay for Type 2 and 97% FPAL and 3% organoclay Type 3, 95% of FPAL and 5% of organoclay for Type 4, and 90% FPAL and 10% organoclay for Type 5 [3] were compared together in terms of mechanical and physical properties as potential of landfill liner.

B. Theory

Geomembranes on slopes experience tension when the frictional forces on it are unbalanced. Geomembrane thickness is related to the pressure exerted upon it. The waste materials, which are placed on landfill liner slopes, tend to slide down because of the weight. Tension (T) in geomembrane is a function of weight of waste above the geomembrane (W), mobilized upper and lower interface friction angles (δ_u, δ_l) between geomembrane and soil layer and also slope angle (β); as shown in (1) [17].

$$T = f(W, \delta_u, \delta_l, \beta) \quad (1)$$

The tension in geomembrane is calculated by (2) [18].

$$T = \sigma_{all} \times t \quad (2)$$

where T is the tension mobilized in the geomembrane and β is slope angle of geomembrane in landfill. The value of σ_{all} is determined by tensile test. The tension is resolved into its horizontal components, which must be resisted by the shear forces. In addition, the vertical component which is assumed to be dissipated along the mobilization distance x must be added to the normal stress imposed by the overlying solid waste (and soil, if applicable). These relation as shown in (3) and (4) and T can be simplified to (5) [18].

$$T \cos \beta = F_{\sigma_u} + F_{\sigma_l} + F_{\sigma_T} \quad (3)$$

$$T \cos \beta = \sigma_n \tan \delta_l(x) + 0.5(2T \sin \beta / x) \times x \tan \delta_l + \sigma_n \tan \delta_u(x) \quad (4)$$

where F_{σ_u} and F_{σ_l} are friction forces between upper layer and lower layer with geomembrane and soil respectively, and F_{σ_T} is the normal stress imposed by the friction angle of overlying solid waste (upper layer).

$$T = \frac{\sigma_n x (\tan \delta_u + \tan \delta_l)}{\cos \beta - \sin \beta \tan \delta_l} \quad (5)$$

From (2) and (5) we have the general formula which is used for calculating thickness of liner that as shown in Equation (6) [18].

$$t = \frac{\sigma_n x (\tan \delta_u + \tan \delta_l)}{\sigma_{all} (\cos \beta - \sin \beta \tan \delta_l)} \quad (6)$$

The magnitude of tension and thickness that were obtained from solving the (5) and (6) by Mathcad software were used to modeling the geomembrane liner in ANSYS software. The model was simulated under solid structural analysis of ANSYS software. The distribution of equivalent stress output from ANSYS was studied and the magnitude of critical zones in which maximum stress was occurred was discussed and validated by tensile stress of materials. The Von Mises criterion is a formula for combining the stresses that these stresses are as result of tension forces; into an equivalent stress. The conventional approach for evaluating Von-Misses equivalent stress was indicated as follow (7):

$$\sigma_{eq} = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(T_{xy}^2 + T_{yz}^2 + T_{zx}^2)] \quad (7)$$

where $\sigma_x, \sigma_y, \sigma_z$ are normal stresses in x, y, z plane respectively and T_{xy}, T_{yz}, T_{zx} are shear stress in respectively xy, yz, zx plane.

C. Computer Simulation and Analytical Method

The theoretical equations are used for calculating the thickness of landfill liner. The optimum values of thickness were used to develop the geometry of landfill in ANSYS version 11, and the mechanical properties of materials assigned to the liner material. Further, symmetric boundary conditions were applied along the edges of liner through the coordinate system in three directions (i.e. X, Y, Z). Symmetric models offered an impressive improvement in run time of calculation with Ansys software [19] and also it is a scale of real condition in landfill liner. Finally the exerted forces (i.e. Normal force and friction force due to the weight of waste and friction between liner and soil) were applied.

The Finite Element Methods (FEMs) are widely used to understand the static and dynamic behavior of various systems. ANSYS structural mechanics solutions have the ability to simulate every structural aspect of a product, including linear static analyses that simply provide stresses or deformations. Mathcad is mathematical engineering software for calculations by combining equations, text and graphics in a presentable format. Fig. 1 indicates the flow chart of procedure that proposed for investigation in appropriate thickness with respect to experimental results and analytical computer simulation method.

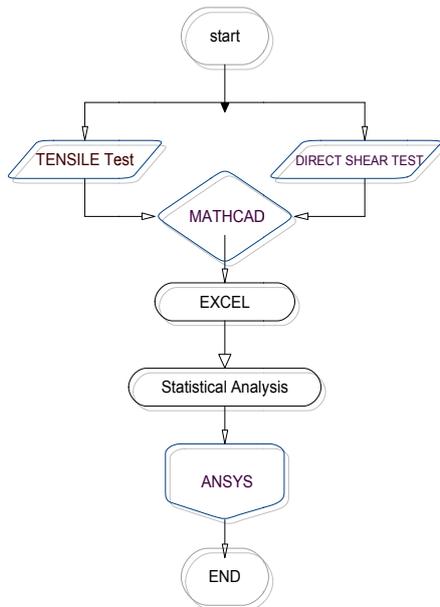


Fig.1. Schematic diagram of methodology

D. Tensile Test

Based on [20] tensile test for five types of recycled material and HDPE were conducted. Fig. 1 shows the size and shape of the specimens based on type IV according to [20] (Fig. 2)

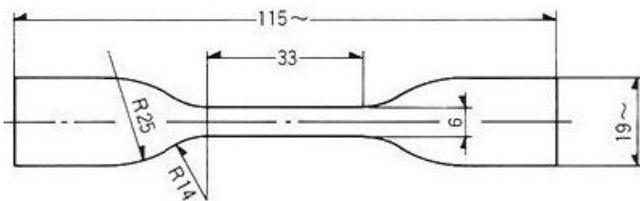


Fig. 2. Size and shape of the specimen [19]

According to [21] direct shear box was used for evaluating soil-to-geomembrane friction angle. This test method covers a procedure for determining the friction angle of geosynthetic against soil, under a constant rate of deformation with different normal load. Two types of soil (clay, sand) were used for determination of friction angle. A clay liner consists of one or more layers of cohesive soil that has been compacted to achieve a low permeability. The purpose of a clay liner as lower layer of geomembrane is to serve as a barrier between waste materials and the environment by limiting seepage from the landfill [22]. The upper (primary) geomembrane liner usually functions to collect the leachate [23]. The sand liner is upper drainage layer and it is used as a leachate collection system.

III. RESULTS AND DISCUSSION

A. Tensile Strength Results

For six specimens of each recycled material types and HDPE, with 5 (mm/min) speed rate and at temperature 23 (C) and 50(%) relative humidity test was conducted by Instron test machine. Table I shows the details of tensile test for five recycled material types and HDPE.

TABLE I: TENSILE STRENGTH, MODULUS AND ELONGATION FOR RECYCLED TYPEES AND HDPE

Sample	Tensile strength (Mpa)	Modulus of elasticity(Mpa)	Elongation at break (%)
Type 1	6.98	262	12.07
Type 2	7.21	617	19.09
Type 3	7.47	654	12.98
Type 4	8.73	839	19.01
Type 5	8.73	165	13.78
HDPE	25	930	678

Results show that in comparison between recycled types in tensile strength Type 4 has the highest tensile, modulus and elongation among other types. Further HDPE has a big difference in tensile and elongation in comparison to recycled type and tensile strength of this material is much higher than five types.

B. Direct Shear Box Test Results

According to [21] direct shear tests were conducted for recycled types and HDPE. Based on [21] when the geosynthetic material is to be tested in the wet condition, the specimens must be soaked in water for a minimum of 24 hours before the test. Hence, all samples were immersed into the leachate (i.e. similar to condition in landfill) for 48 hours before performing the test. In direct shear test, specimen (i.e. 5 cm × 5 cm) and one contact surfaces (i.e. clay or sand), were placed within a direct shear box. The specimens are hydrated, consolidated, and placed under three different constant normal loads in accordance with the ASTM procedures. Table II indicates the friction angle for types and HDPE.

TABLE II: INTERFACE FRICTION ANGLE OF RECYCLED TYPES AND HDPE WITH CLAY AND SAND

Sample	Organoclay (%)	Friction Angle (clay) degree	Friction Angle (sand) degree
Type 1	0	20	18
Type 2	1	20	19
Type 3	3	23	19
Type 4	5	24	20
Type 5	10	26	20
HDPE	0	18	16

As shown in Table II Type 1(0% organoclay) has the lowest friction angle and Type 5 (10% organo clay) has the highest friction angle for clay among other types and HDPE. Therefore this difference between friction angles value can be due to associated with organo clay that has effected in morphology property of interface between material and soil. The following Fig. 3, 4 and 5 show the scanning electron microscopy (SEM) of Type 4, 5 and HDPE respectively.

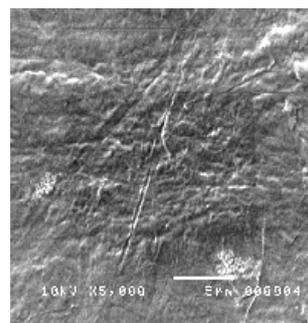


Fig. 3. HDPE

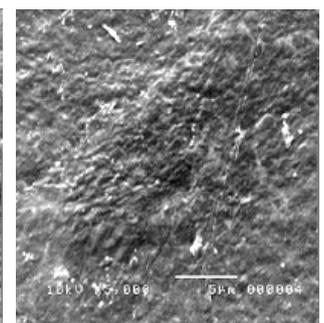


Fig. 4. Type 2

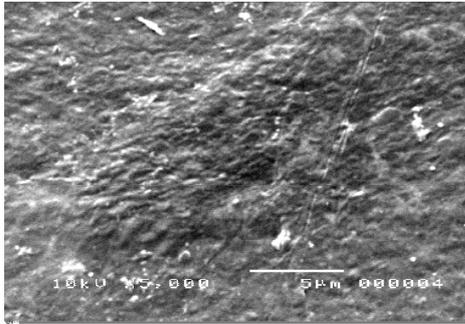


Fig. 5. Type 5

As shown in Fig. 3 and 5 it can be seen that Type 5 that have a 10 percentage of organo clay have more rougher surface in comparison to Type 2 with 1 percentage of organoclay. In addition, the use of smooth geomembrane will lead to low interface shear capacity between landfill liner components which can be considered as one of the major factors in the landfill slope stability failures[12].

Results of experimental tests were used to calculation tension and thickness base on (5) and (6) respectively. Mathcad program version 15 was used to calculation the governing equations (6) in respect to the assumed site specification in landfill. Table III shows input data and assumed site condition for using in Mathcad and analysis procedure.

TABLE III: INPUT AND ASSUMED DATA FOR ANALYSIS PROCEDURE

condition	Data range	HDPE	Type 5	Type 4
Height (h)	10-30 (m)	6	6	6
Slope angle (β)	10-45 (degree)	7	7	7
Unit weight	10-17 (kN.m ⁻³)	14*	14*	14*
Mobilize distance (x)	50-300 (mm)	6	6	6
Tensile strength	6980-25000 (KN.m ⁻²)	25000*	8730*	8730*
Friction angle (clay)	18-26 (degree)	18*	26*	24*
Friction angle (sand)	16-20 (degree)	16*	20*	20*

*Exact values

Data range column shows the acceptable range of variables for different operating conditions. Since the capacity is in a relation with the height of waste fill in the landfill, 6 different sets among the range are analysed. The mobilized distance depends on applied normal stress. Therefore, it changes with the height of over burden material and the data range for geomembrane was assumed in [18]. The interface friction angle of geomembrane-clay and interface friction angle of geomembrane-sand are obtained from experimental results.

The slope angle of landfill contributes in tension and stability in geomembrane. Unit weight of waste has different values depending on the composition of waste materials and data range was obtained from [24] and exact value was selected based on unite weight of PAYATAS landfill in the Philippines [24]. Further, Type 4 and 5 that have a higher tensile strength and friction angle among other types were selected for following analysis.

C. Tension Results

Based on (5) and Table III tension was calculated by Mathcad for Type 5 (i.e. with highest friction angle among

other types), Type 4, and HDPE. Table IV shows the data range tensile results for the same site condition (i.e. height and slope angle) of these materials.

TABLE IV: TENSILE DATA RANGE

Sample	Tension (KN/m)	Changes
Type 4	6-259	253
Type 5	6-303	297
HDPE	4-161	157

As shown in Table IV, tension changes in Type 5 were obtained greater than other materials. Based on (2), and (1) it is due to effect of higher friction angle that belongs to Type 5(see Table II) that has resulted in increasing of tension forces and corresponding thickness. By using safety factor (FS) for cover soils on geomembrane liner; acceptable data range for friction angle and slope angle is determined. Reference [4] shows the design method for determining safety factor that used to assess the stability of cover soils based on static conditions in landfill. Equation (8) shows relation between safety factor, slope angle (β), and interface friction angle (δ) between cover soil and geomembrane [4].

$$FS = \frac{\tan(\delta)}{\tan(\beta)} \quad (8)$$

Fig. 6 shows the results of safety factor was calculated based on (7) and Table III.

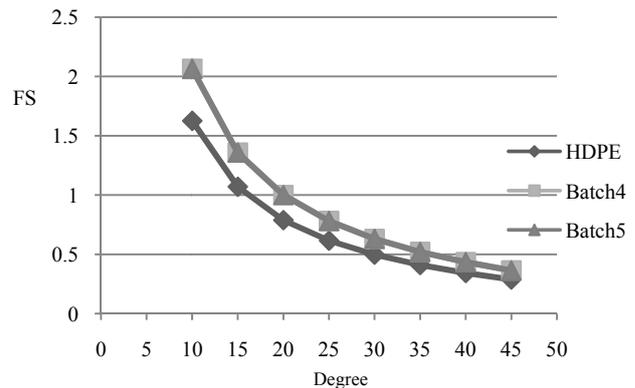


Fig. 6. Fc Vs. Slope Angle For Type 4, 5, And HDPE

As shown in Fig. 6 for slope angles that corresponding FS is equal or greater than 1 are included acceptable range. Therefore for Type 4, 5 and HDPE the slope angle was obtained 20, 20, and 16 degree respectively.

D. Optimum Results

According to assumption and input data in Table III by solving (6) with Mathcad software 289 results with changing condition and composition were obtained. Hence, optimum values of thickness for each types and HDPE were evaluated by using Excel software and insert the results from Mathcad into excel worksheet and filtering the columns of data by imposing simple constraints such as: minimum thickness (i.e. due to reduce the consumption material in liner) in the interval common range between 1mm to 3mm [14], higher slope angle with consider to limited slope angle (See Fig. 4) and also higher height of the solid waste to have more landfill capacity. Table V shows the results of optimum thickness and corresponding site conditions.

TABLE V: OPTIMUM THICKNESS RESULT

Sample	H (m)	X (mm)	β (degree)	Thickness (mm)	Thickness Range (mm)
HDPE	30	100	16	1.2	0.18-6.45
Type 5	18	100	20	3.5	0.77-34.72
Type 4	18	100	20	3.3	0.71-29.77

As shown in Table V the optimum thickness for HDPE was obtained 1.2 mm that is smaller than other types. This is due to effect of higher mechanical property (i.e. tensile strength) and lower tension force (see Table IV) for HDPE in comparison to recycled types. Further, in comparing between optimum thickness of Type 4, and 5 it can be seen there is a small difference between two types. Also, tensile strength for Type5, and 4 were obtained same value. Therefore, this small change in thickness has been affected by tension force and corresponding friction angle (See (2), and Table II for clay). In addition, by using correlation analysis method effect of site condition (slope angle, and height) in thickness were measured. A Pearson correlation is a number between -1 and +1 that measures the degree of association between two variables. Table VI shows the results of correlation analysis for Recycled types and HDPE with the two site condition as variables.

TABLE VI: CORRELATION COEFFICIENT OF VARIABLE

Thickness	Height	Slop angle
Type 4	0.484	0.387
Type 5	0.473	0.417
HDPE	0.427	0.206

Table VI shows that effect of height of overburden waste in landfill on geomembrane for type 4 is higher than other types and HDPE. Also as mentioned above in comparison between thickness of Type 4, and 5 it can be seen based on (6) with increasing in height and slope angle thickness increases. Hence, for type 4 with higher height and lower slope angle correlation coefficient (i.e. 0.484 and 0.387 respectively) has resulted in lower thickness. Further, Table VI shows that site condition has lower effect in HDPE among the other types.

By using a multiple linear regression model relation between the thicknesses (t) and other variables (H, x, β) were obtained. The calculation was performed by using Stat Graphic version 15 program. Equations (9), (10), and (11) show the regression model for HDPE, Type 5, and 4 respectively.

$$t = -3.12791 + 0.0092044 * X + 0.0328967 * \beta + 0.0903692 * H$$

(Mean absolute error = 0.285749) (9)

$$t = -2.15886 + 0.0411431 * \beta + 0.0184152 * X + 0.0959267 * H$$

(Mean absolute error = 0.322979) (10)

$$t = -2.00742 + 0.0351096 * \beta + 0.0176931 * X + 0.0919144 * H$$

(Mean absolute error = 0.345907) (11)

As shown above HDPE has the lowest coefficient for three variables with the highest absolute constant value among the other types. The constant value shows the effect of other parameters such as tensile strength in thickness. Therefore it

shows Effect of site condition in HDPE is less than other types and it is in agreement with results of Pearson correlation analysis.

The optimum values of three samples were used to develop the geometry of landfill in ANSYS version 11 program, mechanical properties of each samples were assigned to the geometry in different site condition. Results of optimum thickness were used to modelling the landfill geometry for structural analysis in ANSYS version 11. Table VII shows the input data for ANSYS software.

TABLE VII: INPUT DATA IN ANSYS

Sample	Tension in slope($\beta=20$), N	Tension in Zero slop ($\beta=0$), N	Tensile Strength (Mpa)	Young Modulus (Mpa)	Height of Waste (m)	Slop angle (degree)
Type 4	31652	24923	8.73	839	22	20
Type 5	33941	26232	8.73	165	22	20
HDPE	29133	25690	25	930	30	15

Fig. 7, 8, and 9 show the ANSYS results for the contour plot maximum equivalent stress of Type 5, 4, and HDPE respectively.

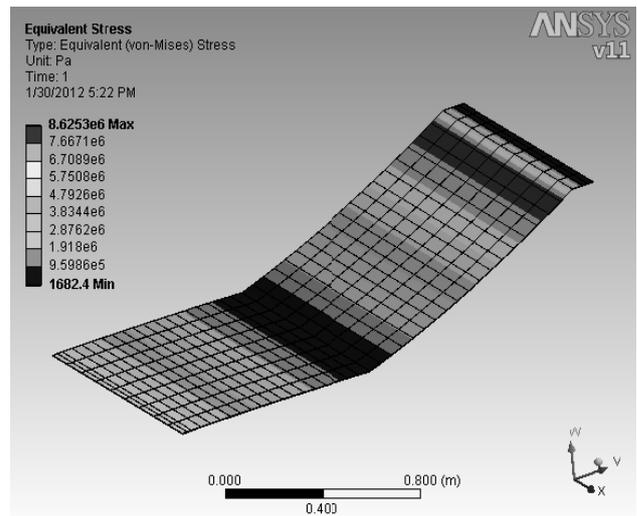


Fig. 7. The contour plot equivalent stress of type 5

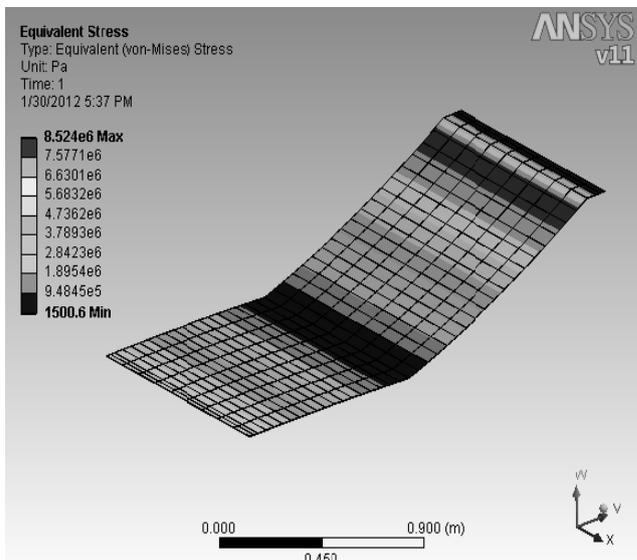


Fig. 8. The contour plot of equivalent stress of type 4

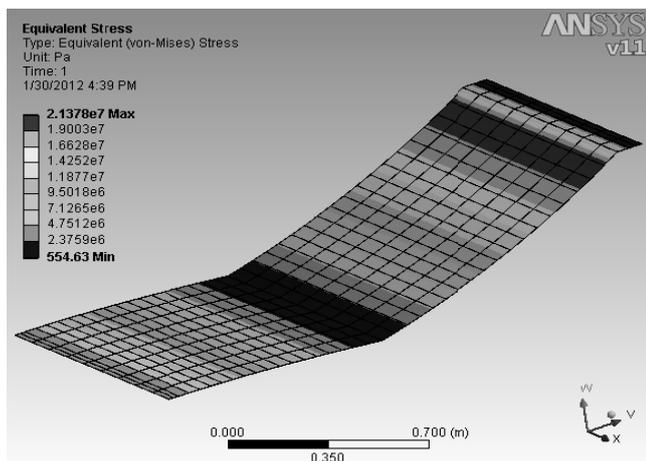


Fig. 9. The contour plot of equivalent stress of hdpe

As shown in contour plot of equivalent stress distribution developed in ANSYS, it can be seen that as anticipated the maximum stress is experienced at the highest part of the slope on upper layer of liner with the value of 8.62 MPa, 8.52 Mpa, and 21.37 Mpa for Type 5, 4, and HDPE respectively.

The upper points should bear accumulate tensile strength that exerted at the lower levels. It means by increasing the height of waste and therefore increasing of normal force the stress in upper level of liner will increase (Geomembrane liner was fixed at the top point) and maximum value will reach at the highest point. Table VIII shows the maximum equivalent stress output results of ANSYS with corresponding tensile strength for each material.

TABLE VIII: MAXIMUM EQUIVALENT STRESS AND TENSIL STRENGTH

Sample	Maximum Equivalent stress (Mpa)	Tensile Strength(Mpa)
HDPE	21.37	25
Type 4	8.52	8.73
Type 5	8.62	8.73

Table VIII shows that all of the equivalent stress for all material are smaller than tensile strength and it shows the optimum results of material were validated.

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