On the Numerical Exploration of Zero-Dimensional Greenhouse Model using Newton-Raphson and Steepest Descent Methods

Rahmat Riza, B. Ariwahjoedi, and Shaharin A. Sulaiman

***Abstract*— Study has showed that Greenhouse gases are the main contribution to global warming. Climate modeling is one of method to explain the influence of those gasses to global warming. Many climate modeling have been proposed to analyze global warming process. Among them, Zero-Dimensional model as the simple model is able to generally describe the influence factors of the climate process. It is important to know the characteristic of each parameter by re-examining the current model. The model derived by Boeker-Van Grondelle offers simple description of global climate processes occurred. The model is system equations consists of two equations and variables. The systems are being studied by using two numerical methods, namely: *Multiple-Equation Newton-Raphson* and Steepest Descent Method. The accuracy of both methods was analyzed to obtain the best solution to the system equations. The proposed solution was verifying by the result that previously published.**

***Index Terms*—Climate modeling, numerical methods, Newton-Raphson and Steepest Descent.**

I. Introduction

The Rise of earth temperature as the result of greenhouse gases (GHGs) effect resulted obviously uncomfortable to the human being. This process is influenced by a few minor gases, namely: carbon dioxide (CO_2), methane, water vapor and ozone. These minor gases trap energy from the sun. This process is shown on Figure 1.

The presence of CO_2, methane, water vapor and ozone in determined concentration are useful to warm the earth by about 33^{o}C in order to be habitable for human being. Without these gases, Earth's average temperature would be about 60^{o}F colder. So, increasing the amount of these gases should effect the average temperature overall. In 20th century, the global-average temperature has increased for 0.7^{o}C [1]. The increasing of the temperature has shown relation to the increasing of GHGs especially CO_2. Climate models have been developed in order to study the relation.

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Rahmat Riza is a lecturer at AT PIRI, Kopertis V, Yogyakarta, Indonesia. He is currently a PhD student at the Mechanical Engineering Department, Universiti Teknologi PETRONAS, Malaysia (email: r_riza_p@yahoo.com).

Dr. Bambang Ariwahjoedi is currently an Associate Professor at the Mechanical Engineering Department, Universiti Teknologi PETRONAS, Malaysia (email: bambang_ariwahjoedi@petronas.com.my).

Dr Shaharin Anwar Sulaiman is currently an Associate Professor at the Mechanical Engineering Department, Universiti Teknologi PETRONAS, Malaysia (email: shaharin@petronas.com.my).

Some models have been proposed to give better description of the climate processes. The most complex models are not as best model since there are many restrictions regarding those models. Zero and one dimensional models are still considering to be solved in order to study although these models are categorized as the simple one.

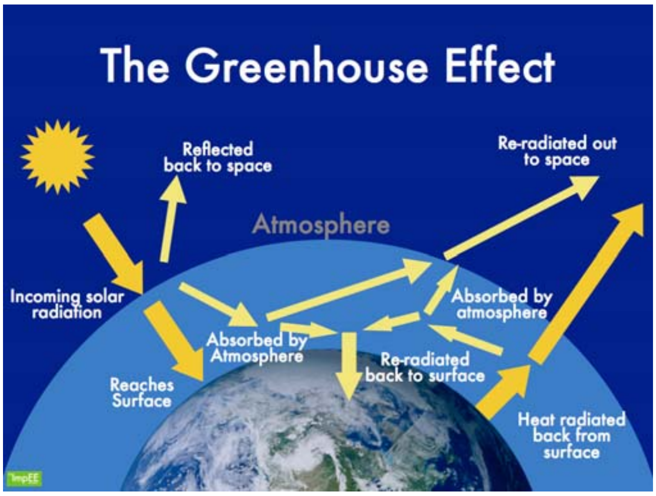

Fig. 1. The greenhouse effect simulation.[2]

On the simple way they could give general explanation to the process occurred between atmosphere and surface of the earth[3]. On this condition, none of dimensional is use neither time nor zonal dimensional. This description is known as Zero-Dimensional model.

Basically, the climate models were mathematical representation of physical processes. The equation developed based on Energy Balance Models (EBM). This model describes that the energy source of the earth is obtained by solar radiation. This energy influences the global temperature of the earth's surface. On the steady state condition, the model is given as equation (1)[3].

$$(1 - a)\pi R^2 S = 4\pi R^2 \sigma T^4 \quad (1)$$

This equation shows the influence of solar radiation to the mean surface temperature of the earth.

In this paper, two numerical methods were used to study the model, namely: Newton-Raphson and Steepest Descent methods. Both methods accuracy are evaluated to propose the best solution for the model. This is followed by comparing them with the previous works to verify the results.

The remainder of the paper is structured as follows. Section 2 introduces climate model analysis. Section 3 describes data collection for the model. Section 4 presents numerical computation of system equations. Section 5 discusses initial values selections. Section 6 reports and discusses the solutions for both Newton-Raphson and Steepest Descent methods. Finally, Section 7 contains the conclusions.

II. Climate Model Analysis

Climate model as on equation (1) was modified to accommodate the process occurred between the atmospheric zone and surface of the earth. This is assumed that atmosphere and surface of the earth as two layers that have heat transfer process. All types of the heat transfer process occurred among both layers. Simplified, the system is able to describe as Fig. 2.

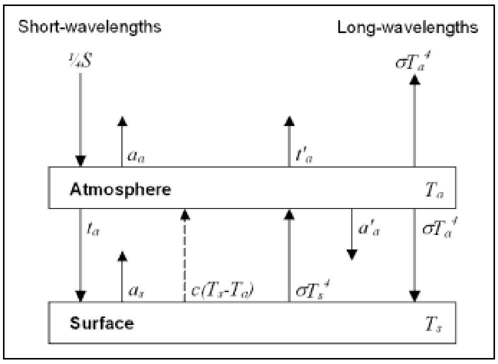

Fig. 2. A zero-dimensional GHG model scheme (Taken from [4]).

The interaction between the atmospheric and surface's earth as given on Fig. 2 could be modeled as system equations, where for the surface of the earth is described by equation (2) as given by Boeker-Van Grondelle [4].

$$(-\tau_a)(1-a_s)\frac{S}{4} + c(T_s - T_a) + \sigma T_s^4(1 - a'_a) - \sigma T_a^4 = 0 \qquad (2)$$

where;

$(-\tau_a)(1-a_s)\frac{S}{4}$ = model for the absorption of surface of the earth

$c(T_s - T_a)$ = model for non-radiation interaction between the atmosphere and surface of the earth.

$\sigma T_s^4(1-a'_a)$ = model for emitted radiation minus the backscattered

σT_a^4 = model for the incoming heat radiation from atmosphere

and mathematical modeling that describes process occurred on atmosphere layer is given on equation (3) [4].

$$-(1 - a_a - \tau_a + a_s\tau_a)\frac{S}{4} - c(T_s - T_a) - \sigma T_s^4(1 - \tau'_a - a'_a) + 2\sigma T_a^4 = 0 \qquad (3)$$

where:

$-(1 - a_s - \tau_a + a_s\tau_a)\frac{S}{4}$ = model for solar absorption of the atmosphere

$-c(T_s - T_a)$ = model for non-radiation interaction

$-\sigma T_s^4(1 - \tau'_a - a'_a)$ = model for absorption of radiation of the earth by the atmosphere

$2\sigma T_a^4$ = model for atmospheric emission

The objective of these system equations is to explain the influence of component of the model to variance surface temperature of the earth. This is caused that the components of this system equations give explanation to describe the correlation to the changing of temperature of surface of the earth.

The above system equations called as system equations with T_s and T_a as the variable. Conditions of this system are system equation with two equations and two variables. Simplifying the system, equation (2) is named as $u(T_s, T_a)$ and equation (3) is $v(T_s, T_a)$. The system were able to be solved by numerical methods

III. Data Collection

The main objective of this work is to analyze numerical method to solve Zero-Dimensional model as proposed by Boeker-Van Grondelle. The model required data which described the average condition of earth processes. Data and sources that used in the calculation are given on Table I.

TABLE I: The Using Data

Component	Source	Data
Solar radiation (S)	McGuffie[3]	1370 W/m^2
Albedo of the atmosphere (a_a)	Boeker&Grondelle[4]	0.3
Albedo of the earth (a_s)	Boeker&Grondelle[4]	0.11
Albedo of the Atmosphere for long-wavelength radiation a'_a	Boeker&Grondelle[4]	0.31
Transmission of the atmosphere (τ) for short-wavelength radiation	Boeker&Grondolle[4]	0.53
Transmission of the atmosphere for long-wavelength τ'_a	Boeker&Grondelle[4]	0.06
Interaction between the atmosphere and earth (c)	Boeker&Grondelle[4]	3.2 W/m^2-K
Stefan Boltzmann constant (σ)	McGuffie[3]	5.67x10^{-8} W/m^2-K^{-4}

Some data as on Table I contain the average assumption of the average condition of the earth. This could result the solution quite far from the measured data.

In this work, the coefficient of interaction between the atmosphere and the earth (τ) is preferred to use 3.2 W/m^2-K^{-4} than 2.7 W/m^2-K^{-4}. However, the 'τ' that equal to 2.7 W/m^2-K^{-4} is used to verifying the result.

IV. Numerical Method for System Equations

The zero dimensional models as given on equation (2) and (3) are system equations that consist of two stimulant equations. The solution for the system equations as this model can be obtained by numerical methods. Two numerical methods are used on this work. They are Newton-Raphson method and Steepest Descent method.

1) Newton-Raphson method

Generally, this method could be described by the following steps[5]:

a. Determine the initial value/guessing, (T_{s_i}, T_{a_i}).
b. Determine matrix Jacobian of system equations for the initial value. Matrix Jacobian is developed as equation (4).

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \tag{4}$$

where:

$$J_{11} = \frac{\partial u(T_s, T_a)}{\partial T_s} \tag{5}$$

$$J_{12} = \frac{\partial u(T_s, T_a)}{\partial T_a} \tag{6}$$

$$J_{21} = \frac{\partial v(T_s, T_a)}{\partial T_s} \tag{7}$$

$$J_{22} = \frac{\partial v(T_s, T_a)}{\partial T_a} \tag{8}$$

c. Calculate determinant of matrix Jacobian as on step 2. Adapted from Chapra and Canale [5], determinant of matrix Jacobian is formulated as equation (9).

$$|J| = \left(\frac{\partial u(T_{s_i}, T_{a_i})}{\partial T_s} \frac{\partial v(T_{s_i}, T_{a_i})}{\partial T_a} \right) - \left(\frac{\partial u(T_{s_i}, T_{a_i})}{\partial T_a} \frac{\partial v(T_{s_i}, T_{a_i})}{\partial T_s} \right) \tag{9}$$

d. Calculate the value of the function for both equations of the initial values as on step 'a', namely: $u(T_{s_i}, T_{a_i})$ and $v(T_{s_i}, T_{a_i})$.
e. Compute the Newton-Raphson solution. Adapted from Capra and Canale [5], the formulas for new guessing Newton-Rapshon are given as equation (10) and equation (11).

$$T_{s_{i+1}} = T_{s_i} \frac{u(T_{s_i}, T_{a_i}) \frac{\partial v(T_{s_i}, T_{a_i})}{\partial T_a} - v(T_{s_i}, T_{a_i}) \frac{\partial u(T_{s_i}, T_{a_i})}{\partial T_a}}{|J|} \tag{10}$$

$$T_{a_{i+1}} = T_{a_i} - \frac{v(T_{s_i}, T_{a_i}) \frac{\partial u(T_{s_i}, Ta_i)}{\partial T_s} - u(T_{s_i}, T_{a_i}) \frac{\partial v(T_{s_i}, T_{a_i})}{\partial T_a}}{|J|} \tag{11}$$

f. Analyze the error. Finished the calculation when the error acceptable otherwise returns to step 1.

2) Steepest Descent method

This method used gradient form to bring to closer to the solution linearly. The procedure for this method can be described as follows[6]:

a. Determine the initial value/ initial guessing.
b. Calculate the value of the function for both equations, $u(T_{s_i}, T_{a_i})$ and $v(T_{s_i}, T_{a_i})$.
c. Compute function of g at the initial values (T_{s_i}, T_{a_i}). Adapted from Faires and Burden [6], the formula for function g is written as on equation (12).

$$g(T_{s_i}, T_{a_i}) = [u(T_{s_i}, T_{a_i})]^2 + [v(T_{s_i}, T_{a_i})]^2 \tag{12}$$

d. Analysis the gradient of the equation at the initial value. Adapted from Faires and Burden [6], It is able to present as on equation (13) (∇g).

$$\nabla g = 2J(u(T_{s_i}, T_{a_i}), v(T_{s_i}, T_{a_i})) \tag{13}$$

e. Analyze alpha value, for α>0, near the initial value in order to obtain the right direction by using the values of z_0 and z. z_0 and z are formulated as on equation (14) to equation (16).

$$z_0 = \sqrt{[\nabla g(T_{s_i}, T_{a_i})]^2} \tag{14}$$

$$z_{T_s} = \frac{1}{z_0} \nabla g(T_{s_i}) \tag{15}$$

$$z_{T_a} = \frac{1}{z_0} \nabla g(T_{a_i}) \tag{16}$$

f. Define the new value that closer to the solution. Adapted from Faires and Burden [6], this is formulated as on equation (17) and equation (18).

$$T_{s_{i+1}} = T_{s_i} - \alpha z_{T_s} \tag{17}$$

$$T_{a_{i+1}} = T_{a_i} - \alpha z_{T_a} \tag{18}$$

where α is the best α obtained on step 'e'.

g. Analyze function 'g' by using new value guessing by using equation (12). The result is used to analyze the new guessing by comparing to the value of function 'g' that uses the previous values as on step 'c'.
h. Do re-iteration whenever the step 'g was not satisfied the condition.

V. Initial Values Selection

There are three probabilities of initial values that were selected on this work, namely as follows:

a. Zero Kelvin. The unit temperature as on data collection based on Kelvin measurement. It should be the first priority to be selected as the first initial values on the numerical solution. This initial value is used too as the testing of the algorithms.
b. Zero degree Celsius. In fact that Zero Kelvin is equal with -273.15 ^{o}C, the guessing value easily considers being located very far from the solution. This caused the zero degree Celsius should be considered as it is equal

to 273.15 K.

c. The average surface temperature of the earth. This is the nearest initial guessing to the solution. However, there is still a distance between them that need to be found.

VI. Result and Discussion

A. Algorithms of the Model

Sample algorithms for both Newton-Raphson and Steepest-Descent methods are given as follows:

a. Newton-Raphson Algorithms
Few commands that used to solve the equations are list as follows:

```
'Jacobian component determinant = partial derivative of the parametrics';
Ts2=Ts1+h
Ts3=Ts2+h
uTs1=(c*(Ts1-Ta1))+(sigma*(Ts1^4)*(1-aldoatlw))-(sigma*(Ta1^4))-(taua
t*(1-aldoes)*(S/4))
uTs2=(c*(Ts2-Ta1))+(sigma*(Ts2^4)*(1-aldoatlw))-(sigma*(Ta1^4))-(taua
t*(1-aldoes)*(S/4))
uTs3=(c*(Ts3-Ta1))+(sigma*(Ts3^4)*(1-aldoatlw))-(sigma*(Ta1^4))-(taua
t*(1-aldoes)*(S/4))
deruTs=(-uTs3+(4*uTs2)-(3*uTs1))/(2*h)
 .
 .
 'Jacobian determinant';
jdet=(deruTs*dervTa)-(deruTa*dervTs)
u11=(c*(Ts1-Ta1))+(sigma*(Ts1^4)*(1-aldoatlw))-(sigma*(Ta1^4))-(tauat
*(1-aldoes)*(S/4))
v11=-(c*(Ts1-Ta1))-(sigma*(Ts1^4)*(1-tauatlw-aldoatlw))-((1-aldoat-tauat
+(aldoes*tauat))*(S/4))+(2*sigma*(Ta1^4))
Ts1=Ts1-(((u11*dervTa)-(v11*deruTa))/jdet)
Ta1= Ta1-(((v11*deruTs)-(u11*dervTs))/jdet)
```

b. Steepest Descent Algorithms
The samples of list command for Steepest Descent algorithms are given below:

```
'Numerical calculation of partial derivative for all equations';
'initial condition';
Ts2=Ts1+h
Ts3=Ts2+h
Ta2=Ta1+h
Ta3=Ta2+h
'calculation for the derivative u on Ts';
uTs1=(c*(Ts1-Ta1))+(sigma*(Ts1^4)*(1-aldoatlw))-(sigma*(Ta1^4))-(taua
t*(1-aldoes)*(S/4))
uTs2=(c*(Ts2-Ta1))+(sigma*(Ts2^4)*(1-aldoatlw))-(sigma*(Ta1^4))-(taua
t*(1-aldoes)*(S/4))
uTs3=(c*(Ts3-Ta1))+(sigma*(Ts3^4)*(1-aldoatlw))-(sigma*(Ta1^4))-(taua
t*(1-aldoes)*(S/4))
deruTs=(-uTs3+(4*uTs2)-(3*uTs1))/(2*h)
 .
 . .
'calculating the new guessing';
Ts1=Ts1-(alpha*zTs)
Ta1=Ta1-(alpha*zTa)
```

B. Algorithms Testing

The model is developed by using Matlab programming. It was tested by manual calculation. On this case, the initial values $\left(T_{s_i}, T_{a_i}\right)$ of zero Kelvin were used. The results for first iteration calculation from both manual calculation and result of algorithms are given as follows:

a. Manual calculation
1. Newton-Raphson :
$T_{s_i} = 5.3187\text{x}10^9$ K ; $T_{a_i} = 5.3187\text{x}10^9$ K
2. Steepest Descent :
$T_{s_i} =$ 6.5125 K ; $T_{a_i} = -$ 6.5125 K

b. The results of algorithms:
1. Newton-Raphson :
$T_{s_i} = 5.3187\text{x}10^9$ K ; $T_{a_i} = 5.3187\text{x}10^9$ K
2. Steepest Descent :
$T_{s_i} =$ 6.5125 K ; $T_{a_i} = -$ 6.5125 K

Results as shown on point ‘a’ and ‘b’ above showed that the developed algorithms have given the answers as it. This means that the algorithms should be running for all the initial values.

Moreover, this results is showed that Steepest Descent method offer the better solution whenever the initial values is very far from the real solution itself. This is able to be seen that the Newton-Raphson gives unrealistic solution for this initial values where the Steepest Descent give acceptable answer to the solution. The worse results of Newton-Raphson on this point were caused by determinant of matrix Jacobian of the procedure was very small, near to zero. This brings the method to deviate far enough form the direction of the real solution.

C. The Results of the Numerical Method for the Zero-Dimensional Model

Result for initial values (273.15 K, 273.15 K) and initial values (288 K, 288K)

a. Newton Raphson
The results of this method for six iterations are given on Table 2 for initial value (273.15 K, 273.15 K) and Table 3 for initial values (288 K, 288 K).

TABLE II: Results for Newton-Raphson Method

Step	Initial values (273.15 K, 273.15 K)		Initial values (288 K, 288 K)	
	T_s	T_a	T_s	T_a
1.	287.5742	251.7691	288.8224	255.8830
2.	285.6987	248.8045	285.8182	249.0358
3.	285.6649	248.7521	285.6652	248.7526
4.	285.6649	248.7521	285.6649	248.7521

Newton-Raphson offered very fast of the iteration process to solve this system equations. Both solution with initial values (273.15 K, 273.15 K) and initial values (288 K, 288 K) could give exactly same direction for point of solution on T_s =285.6649 K and T_a = 248.7521 K.

The solution of this method offer straight forward converge both the starting points of iteration process located smaller and bigger than the solution itself. As they are near to the real solution this method offer very fast solution.

b. Steepest Descent
As previously discussed the value of ∇g as equation (13) were the consideration point to determine the real solution that could be achieved by this method. Steps Iteration results as given by Table III showed that this method takes more steps compare to Newton-Raphson process. It takes almost

two times of iteration steps compare to Newton-Raphson procedures that used same initial values as given on Table II.

TABLE III: RESULTS FOR STEEPEST DESCENT METHOD BY USING INITIAL VALUES (273.15 K, 273.15 K)

Step	α	$T_s(K)$	$T_a(K)$	∇g
1.	27.16	287.0171	249.7969	8.5206
2.	1.2300	286.0979	248.9796	2.3522
3.	0.0800	286.0447	249.0394	0.6605
4.	0.0200	286.0247	249.0394	0.6268
5.	0.0100	286.0249	249.0294	0.5971
6	0.3500	285.7354	248.8327	0.1331
7.	0.0200	285.7466	248.8161	0.0312
8.	0.0100	285.7431	248.8067	0.0308
9.	0	285.7431	248.8067	0.0310

The solution of every iteration step describe that the Steepest Descent do not always go to the nearer point of the real solution. This is given by the result on step 4 and step 5 for the T_s solution. The solution produced on the step 5 afield from the real solution. However, this process was not occurred on solution of T_a. This could be a signal that the gradient of the method used point T_a as the path or based point to find the real solution.

Results of iteration steps by using initial values (288 K, 288 K) are given on Table IV.

TABLE IV: RESULTS FOR STEEPEST DESCENT METHOD BY USING INITIAL VALUES (288 K, 288 K)

Step	α	$T_a(K)$	$T_s(K)$	∇g
1.	30.0400	302.7406	261.8253	1.5588e+003
2.	10.0400	294.0806	256.7453	796.2288
3.	1.2300	294.7029	255.6843	406.3225
4.	4.8500	290.5008	253.2626	227.6850
5.	0.6400	290.8203	252.7081	127.6338
6	2.2400	288.8538	251.6355	82.3760
7	0.3500	289.0213	251.3282	53.2232
8	1.3300	287.8466	250.7044	36.1435
9	0.2200	287.9500	250.5102	24.4648
10	1.8000	286.4341	249.5397	9.0708
11	0.1600	286.5204	249.4050	3.3706
12	0.0900	286.4334	249.3820	3.0677
13	0.0400	286.4438	249.3434	2.7888
14	0.0500	286.3940	249.3387	2.6259
15	0.0300	286.3975	249.3089	2.4664
16	0.0500	286.3479	249.3025	2.3127
17	0.0300	286.3519	249.2728	2.1672
18	0.0400	286.3120	249.2708	2.0495
19	0.0300	286.3142	249.2409	1.9363
20	0.0300	286.2844	249.2441	1.8431
21	0.0200	286.2651	249.2194	1.6793
22	0.0800	286.2223	249.1518	1.4962
23	0.0300	286.1971	249.1682	1.3259
24	0.0400	286.1805	249.1318	1.2349
25	0.0200	286.1622	249.1398	1.1534
26	0.0500	286.1374	249.0964	1.0485
27	0.0200	286.1198	249.1059	0.9620
28	0.0700	286.0801	249.0483	0.8375
29	0.0200	286.0636	249.0595	0.7330
30	0.4800	285.6769	248.7751	0.0297
31	0.0100	285.6828	248.7670	0.0018

Results given by Table IV showed that Steepest Descent methods walk with very slowly rate of convergence to obtain the point solution. This is analyzed from the value of ∇g for each iteration step. The value of ∇g reduced quite little from step to step of the iteration process.

By using this initial value (288 K, 288 K), the Steepest Descent method required large number of step iteration. They are total 31 steps iteration to find solution with acceptable value of ∇g .

As on results of using initial values (273.15 K, 273.15 K), the results of using initial values (288 K, 288 K) showed the trend to deviate from the path of real solution.

D. Verifying the Solution

Verification of the work is done by using the 'τ' is 2.7 $W/m^2\text{-}K^{-4}$ as the coefficient of interaction between the atmosphere and the earth. Applying this number to the system equation and using the same method as discussed above, Newton-Raphson result Ts is equal to 288.3129 K. In other hand the suggested solution, given by Boeker and Van-Grondelle [4] for the same of 'τ' parameter, T_s is around 288 K.

The result of iteration procedure for this method is given as on Table V.

TABLE V: RESULTS FOR NEWTON-RAPHSON METHOD FOR THE VERIFICATION BY USING INITIAL VALUES (273.15 K, 273.15 K)

Step	$T_s(K)$	$T_a(K)$
1.	290.4031	251.5994
2.	288.3483	248.5630
3.	288.3130	248.5078
4.	288.3129	248.5077

The solutions were obtained in four steps iteration. This is same as the previous procedure where the 'τ' parameter equal to 3.2 $W/m^2\text{-}K^{-4}$.

Moreover, Table V shown that there is no changing for the temperature of atmosphere (T_a) compare to the result of the previous computational.

VII. CONCLUSION

Overall, Newton-Raphson method offer the fastest solution to solve Zero-Dimensional model as given by Boeker-Van Grondelle. This is considered of two main parts, namely: speed of convergence and consistency of the results. Newton-Raphson could solve the system equation using 4 to 5 steps of iteration process. It could be done by using both initial values (273.15 K, 273.15 K) and initial values (288 K, 288 K). On this case, Steepest Descent could only do the process twice of the process of Newton-Raphson methods. Moreover, consistency of the Newton-Raphson could be concluded by analyzing the solution offered from both initial values. By using two different initial values, the Newton-Raphson gave same point solution. In other case, Steepest Descent gave different solution from different initial values.

By modifying the parameter of 'τ' and making comparison to the result published by Boeker and Grondelle [4], the solutions have been resulted by the method concluded as the acceptable results.

The results from this calculation describe the average of surface temperature of the earth (T_s) and average temperature of the atmosphere (T_a). This was caused by the parametric used on this system equations were the assumption of the average condition of the earth. In fact, some assumptions were clearly quite far from the real value[4].

However, by comparing the result given on different 'τ', the model showed that it was able to describe the influence of the parameters to the changing of surface temperature.

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Rahmat Riza is a lecturer at the Mechanical Engineering Department, AT PIRI, Kopertis V, Yogyakarta, Indonesia. He is currently a PhD student at the Mechanical Engineering Department, Universiti Teknologi PETRONAS, Malaysia. He received his BS in Mechanical Engineering in 2003 and M.Sc (Mechanical Engineering) in 2009 from Mechanical Engineering Department, Universiti Teknologi PETRONAS, Malaysia. Hir research interest includes energy, modeling, and climate change.

Dr. **Bambang Ariwahjoedi** is currently an Associate Professor at the Mechanical Department, Universiti Teknologi PETRONAS, Malaysia. He received his BS in Chemistry from Bandung Institute of Technology, Masters of Physical Chemistry from Bandung Institute of Technology, Masters of Science in Science and Technology of Ceramics from Sheffield University, and Doctor of Philosophy in Materials Science and Engineering Sheffield from University. His primary areas of research interest involve Bulk and surface characterization of materials, ceramic/oxide-graphitic refractory materials, carbonaceous mesophase and graphitic materials, colloid and interface/surface chemical aspects of ceramics and materials processing, rheology of ceramics slips and green bodies, novel corrosion inhibition approaches, hydrogen production via water splitting, and novel coating approaches.

Dr **Shaharin Anwar Sulaiman** graduated in 1993 with a B.Sc. in Mechanical Engineering from Iowa State University. He earned his M.Sc. in Thermal Power and Fluids Engineering from UMIST in 2000, and Ph.D. in Combustion from the University of Leeds in 2006. During his early years as a graduate, he worked as Mechanical and Electrical (M&E) Project Engineer in a construction company for five years. His research interests include combustion, biomass gasification, sprays and atomization, and air-conditioning & ventilation. He joined UTP in 1998 and at present he is an Associate Professor in the Department of Mechanical Engineering, and he is also the Director of the Energy Research group of the university. He was the Program Manager for the MSc in Asset Management and Maintenance program. Certified as a Professional Engineer with the Board of Engineers, Malaysia, he is also a Corporate Member of the Institution of Engineers Malaysia. Dr Shaharin is a member of the Malaysia Energy Professionals Association (MEPA). At international level, he is a member of the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE).